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(Department of Education).

BRAINSE AN MHEÁN-OIDEACHAIS

(Secondary Education Branch).

LEAVING CERTIFICATE EXAMINATION, 1928.

HONOURS

MATHEMATICS (I).

THURSDAY, 14*th* *JUNE*.—10 A.M. TO 12.30 P.M.

Six questions may be answered.

All questions carry equal marks.

Mathematical Tables may be obtained from the Superintendent.

1. In a right-angled triangle the hypotenuse is a and the perpendicular from the right-angle to the hypotenuse is b. Express the other two sides in the form $\sqrt{X} \pm \sqrt{Y}$ where X, Y are rational functions of a, b.

What condition must be satisfied by a and b so that the triangle may be possible ?

2. Use the Binomial Theorem to find the best approximation to two places of decimals to the value of

$$\frac{(1-x)^{\frac{1}{2}}\ (2+3x)^{-3}}{(1-2x)^{\frac{1}{4}}} \text{ when } x = \cdot 027.$$

3. Factorise :—

(i) $a^3+ab^2+2b^3$,

(ii) $a\,(b-c)\,(b+c-a)^4+b\,(c-a)\,(c+a-b)^4+c\,(a-b)\,(a+b-c)^4$

4. Find, between what limits x must lie so that y may be real if

$$4x^2+2xy+y^2-39x-6y+99=0.$$

Find also the maximum and minimum values of y within these limits.

5. Show that the coefficient of the general term in the expansion of $(1+x)^n$, where n is a positive integer, is the number of combinations of n things taken r at a time.

Find the number of ways a candidate may answer this paper if he answers at least one question and not more than six. A partial answer is to be taken as an answer and the order of the answers is not to be considered.

6. Find a formula for the area of a triangle in terms of the co-ordinates of its vertices. Deduce that $ax+by+c=0$ is the equation of a straight line.

Find the locus of the point whose co-ordinates are $(a+r \cos \theta, b+r \sin \theta)$, where a, b are constant, (i) when θ is constant and r varies, (ii) when r is constant and θ varies.

7. The co-ordinates of the vertices A, B, C of a triangle ABC are $(a, 0)$, $(b, 0)$, $(0, c)$ respectively. Find the equation of the circle which passes through the middle point of AB, the foot of the perpendicular from C to AB, and the middle point of the straight line joining C to the orthocentre. Show that this circle also passes through the mid-points of the other sides.

8. Find the length of the tangent from the point (x_1, y_1) to the circle $x^2+y^2+2gx+2fy+c=0$.

Show that the tangents drawn from any point on the straight line passing through the points of intersection of the circles $C_1=0$, $C_2=0$ to the system of circles $C_1+\lambda C_2=0$ are equal for all values of λ.

9. Find the equation of the locus of the centres of circles which touch a given straight line and a given circle. Show that it is a parabola and find its focus and directrix.