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LEAVING CERTIFICATE EXAMINATION, 1926.

HONOURS

MATHEMATICS (I).

THURSDAY, 17*th* *JUNE*.—10 A.M. TO 12.30 P.M

Tables of Measures, Constants and Formulae, and Logarithmic Tables may be obtained from the Superintendent.

Six questions may be answered. 3 (a) *or* 3 (b) may be attempted, but not both.

All questions carry equal marks.

1. Show that the equation

$$(x+a)^{2n}+(x+b)^{2n}=c,$$

where n is a positive integer, can be reduced to an equation of the nth degree by substituting

$$\sqrt{u}+\sqrt{v}=x+a \quad \text{and} \quad \sqrt{u}-\sqrt{v}=x+b.$$

Solve the equation $x^4+(x+1)^4=97$.

2. Prove that

$${}^{n}C_{r}+{}^{n}C_{r-1}={}^{n+1}C_{r}.$$

Hence or otherwise prove the Binomial expansion for $(x+a)^n$ where n is a positive integer.

3. (*a*) What is meant by a convergent series ?

Show that $u_1 - u_2 + u_3 - u_4 + \ldots$ is convergent if u_1, u_2, etc., are positive and decreasing without limit.

When is $1 + 2x + 3x^2 + \ldots$ convergent ?

or

(*b*) The terms of the series $u_0, u_1, u_2, u_3 \ldots$ are connected by the relation

$$u_n = \frac{(a+2)\,u_{n-1} - 4}{u_{n-1} + a - 2}$$

Show that $$\frac{1}{u_n - 2} = \frac{1}{a} + \frac{1}{u_{n-1} - 2}$$

and hence that $$u_n = \frac{u_0\,(2n+a) - 4n}{nu_0 - (2n-a)}$$

and that $\underset{n \to \infty}{Lt}\, u_n$ is independent of u_0.

4. What are the conditions that $ax^2 + bx + c$, where a, b, c are real, should always be positive for all real values of x ?

Show that $\dfrac{(ax-b)\,(dx-c)}{(bx-a)\,(cx-d)}$ may have all real values, provided that $(a^2 - b^2)\,(c^2 - d^2) > 0$.

5. Prove that

$$\frac{\lfloor n}{\lfloor r \, \lfloor n-r} \left(\frac{x}{n}\right)^r = \left(1 - \frac{1}{n}\right)\left(1 - \frac{2}{n}\right) \ldots \left(1 - \frac{r-1}{n}\right)\frac{x^r}{\lfloor r}$$

and, m being a positive integer, that $\left(1 + \dfrac{1}{m}\right)^m$ increases as m increases.

6. Find an equation representing *any* line through the foot of the perpendicular from the point $(p,\ q)$ on the line $ax + by + c = 0$.

Find the equation to the straight line passing through $(-2,\ 5)$ and through the foot of the perpendicular from $(4, -3)$ to the line

$$3x + 2y - 13 = 0.$$

7. Find c so that the straight line $y = mx + c$ should touch

(i) the circle $x^2 + y^2 = a^2$

(ii) the parabola $y^2 = 4bx$.

If $a = 2b$, find the common tangents to the circle and parabola.

8. Give an equation representing *any* circle through the intersections of the circles

$$x^2 + y^2 - 2x - 6y - 6 = 0$$

and

$$x^2 + y^2 - 4x - 8y + 2 = 0$$

and determine the equation of that one which has the common chord for diameter.

9. (x_1, y_1), (x_2, y_2), (x_3, y_3) are the vertices of a triangle and a, b, c, the lengths of the sides: show that the in-centre of the triangle is at

$$\left(\frac{ax_1 + bx_2 + cx_3}{a + b + c}, \frac{ay_1 + by_2 + cy_3}{a + b + c}\right)$$