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LEAVING CERTIFICATE EXAMINATION, 1925

HONOURS.

MATHEMATICS (II).

FRIDAY, 19*th JUNE*.—Morning, 10 *a.m. to* 1 *p.m.*

(Tables of Measures, Constants and Formulae, and Logarithm tables may be obtained from the Superintendent.)

1. If $y = \frac{u}{v}$, where u and v are functions of x, find the derivative of y in terms of the derivatives of u and v.

(*a*) Find the derivatives of $\frac{x^2}{\cos x}$ and $x^2 \cos x$.

2. (*a*) Show that

$$\frac{(x+h)^{-\frac{1}{2}} - x^{-\frac{1}{2}}}{h} = -\frac{1}{x^{\frac{1}{2}}(x+h) + x(x+h)^{\frac{1}{2}}}.$$

Hence find the derivative of $x^{-\frac{1}{2}}$.

Would there be any possible objection to the use of the binomial theorem in finding the derivative?

(*b*) Find the minimum value of $2x^{\frac{1}{2}} + x^{-\frac{1}{2}}$.

3. A vessel is in the form of a hollow cone, with axis vertical and vertex downwards. The vertical angle of the cone is 2α. Water is poured in at a uniform rate of v cubic feet per minute; at what rate is the level of the water rising in the vessel when the depth of the water is x feet?

4. $$F(x) = 1 - \frac{x^2}{1.2} + \frac{x^4}{1.2.3.4} - \cos x.$$

Write down $F'(x)$, $F''(x)$, $F'''(x)$, $F''''(x)$, i.e. the successive differential coefficients of $F(x)$. Show by reasoning from the last that each is positive between the values $x = 0$, and $x = \frac{\pi}{2}$.

Deduce that between these limits for x,

$$1 - \frac{x^2}{1.2} + \frac{x^4}{1.2.3.4} > \cos x > 1 - \frac{x^2}{1.2}.$$

5. Three towns A, B, C connected by straight roads are at the vertices of a triangle. The distances AB, BC, CA are 12 miles, 15 miles, and 17 miles respectively. A man starts to walk from A to B and thence to C at 3·5 miles per hour, and another at the same time starts from B to walk to C and thence to A at 4 miles per hour. When will the line joining their positions be (a) at right angles to AB, (b) parallel to AB? When before the first man reaches B, will they be nearest together?

6. AB is a chord of a circle, and E is the other end of the diameter through A. EB produced meets the tangent at A in D. Show that $AE^2 - BE^2 = BE \,.\, BD$.

Hence show that as B approaches A along the circumference, the difference $AE - BE$ approaches $\frac{BD}{2}$.

Show that if AB is a side of a regular inscribed polygon of a very large number of sides, the difference between the perimeter of the inscribed polygon and that of the circumscribed polygon of the same number of sides is $\frac{\pi}{2}\,BD$ approximately.

Show that this difference when the radius of the circle is 4,000 miles, and the number of sides 1,000,000, is about $\frac{1}{400}$ in.

7. *ABCD* is a cyclic quadrilateral. *Z* is the intersection of its diagonals, and *X* and *Y* are the intersections of the opposite sides produced. Show that each vertex of the triangle *XYZ* is the pole of the opposite side.

How would you draw a tangent to a circle from a point outside, using a ruler only ?

8. To any given triangle circumscribe the maximum equilateral triangle.

9. *AP*, *BQ*, *CR* are the perpendiculars from the vertices of a triangle *ABC* on the sides. *PD* and *QG* are perpendiculars to *AB*; *PE* and *RH* perpendiculars to *AC*; and *RK* and *QF* perpendiculars to *BC*, the points *D*, *G*, *E*, *H*, *K*, *F* being in the sides of the triangle. Show that the hexagon *DGHEFK* has the following properties :—

(1) The diagonals *DE*, *FG*, *HK* are equal.

(2) The opposite sides are parallel.

(3) The hexagon is inscribable in a circle

(4) This circle is concentric with the circle inscribed to the triangle formed by the diagonals of the hexagon.