Coimisiún na Scrúduithe Stáit
State Examinations Commission

JUNIOR CERTIFICATE EXAMINATION, 2005

MATHEMATICS – HIGHER LEVEL

PAPER 1 (300 marks)

THURSDAY, 9 JUNE – MORNING, 9:30 to 12:00

Attempt ALL questions.

Each question carries 50 marks.
Graph paper may be obtained from the superintendent.

The symbol \( \rightarrow \) indicates that supporting work must be shown to obtain full marks.
1. (a) \( U \) is the universal set. \( P \) and \( Q \) are two subsets of \( U \).

Copy the Venn diagram into your answerbook and shade in the set \((P \cup Q)'\).

(b) (i) Light travels at a speed of approximately \((2 \cdot 9 \times 10^5)\) km/sec.

How many kilometres will light travel in 8 minutes?
Express your answer in the form \(a \times 10^n\), where \(n \in \mathbb{N}\) and \(1 \leq a < 10\).

(ii) A tourist paid $4620 to a travel agent for a holiday in Ireland, where €1 = $1 \cdot 32.

The cost to the travel agent of organising the holiday was €2985.
Calculate, in euro, the profit made by the travel agent.

(c) (i) By rounding to the nearest whole number, estimate the value of

\[
\left(\frac{5.9 + \sqrt[3]{27.24}}{3.06}\right)^2
\]

Then, evaluate \(\left(\frac{5.9 + \sqrt[3]{27.24}}{3.06}\right)^2\), correct to two decimal places.

(ii) Simplify \(\sqrt{3}(2\sqrt[6]{6} - 4\sqrt{3}) - \sqrt{10}(3\sqrt{5} - 2\sqrt{10})\), without the use of a calculator.

Express your answer in the form \(a + b\sqrt{2}\), where \(a, b \in \mathbb{Z}\).
2. (a) (i) Write down the reciprocal of \(\frac{7}{2}\).

(ii) Find the value of this reciprocal, correct to 2 decimal places.

(b) (i) There are 25 000 fish in a fish farm.
The number of fish in the farm increases by 40% each year.  
\[\text{How many fish will be in the farm at the end of 3 years?}\]

(ii) The monthly line rental on Peter’s mobile phone amounts to €12.70.
During May, the duration of his calls is 1 hr 41 mins and 50 secs.
Calls are charged at 0.6 cent per second.

\[\text{Calculate Peter’s total bill for May.}\]

(c) (i) The standard rate of income tax is 20% and the higher rate is 42%.
Sheila has tax credits of €2700 for the year and a standard rate cut-off point of €22,000.
Sheila has a gross income of €45,000 for the year.
\[\text{Calculate the total tax payable by Sheila for the year.}\]

(ii) Tony pays tax at the same rates as Sheila.
Tony has tax credits of €2900 for the year and has the same standard rate cut-off point as Sheila.
His total tax payable amounts to €13,680 for the year.
\[\text{Calculate Tony’s gross income for the year.}\]
3. (a) Write \( \sqrt[3]{16} \) in the form \( 2^k \), \( k \in \mathbb{Q} \).

(b) (i) Factorise \( 3x^2 + 8x - 3 \).

(ii) Factorise \( 3p - c + 3pc - c^2 \).

(iii) Simplify \( (2x - 1)^2 - (x - 1)^2 \).

(c) A box of drinking chocolate powder costs €3 · 60.

(i) If the box contains \( x \) grams of powder, write an expression in \( x \) to represent the cost of 1 gram of the powder.

During a promotion, the manufacturer adds in to the box an extra 30 grams of powder. The cost of the box of drinking chocolate remains at €3 · 60.

(ii) Write an expression in \( x \) to represent the cost of 1 gram of the powder during the promotion.

Each gram of powder, in this case, now costs 1 cent less.

(iii) Write an equation in \( x \) to represent the above information.

(iv) Solve this equation to find how many grams of powder are in the box during the promotion.
4. (a) \( \Rightarrow \) Let \( f \) be the function \( f : x \rightarrow x^2 + x - 7, \ x \in \mathbb{R} \).

Find \( f(-3) \).

(b) Helen buys stamps costing 48 cent and 60 cent.
She buys a total of 50 stamps costing \( 25.68 \) €.

(i) Taking \( x \) to be the number of 48 cent stamps and \( y \) to be the number of 60 cent stamps, write down two equations in \( x \) and \( y \) to represent this information.

(ii) \( \Rightarrow \) Solve the equations to find the number of each type of stamp that Helen has purchased.

(c) (i) \( \Rightarrow \) Express in its simplest form:

\[ \frac{1}{x - 1} + \frac{1}{x + 1}. \]

(ii) \( \Rightarrow \) Hence, or otherwise, solve the equation:

\[ \frac{1}{x - 1} + \frac{1}{x + 1} = 3. \]

Express your answer in the form \( a \pm b\sqrt{10}, \) where \( a, b \in \mathbb{Q}. \)
5. (a) Seven shirts and two sweaters cost €202.50.
A sweater costs the same as four shirts.
Find the cost of one shirt.

(b) In a school of 430 students, 250 students study History, 240 students study Geography.
Let $x$ represent the number of students who study neither History nor Geography.
The number of students who study both History and Geography is 3 times the number who study neither of these subjects.

(i) Represent this information on a Venn diagram.

(ii) Write down and simplify an expression in $x$ for the total number of students in the school.

(iii) Use this expression to find the number of students who study neither History nor Geography.

(c) Let $f$ be the function $f : x \rightarrow x^2 + bx + c$, $x \in \mathbb{R}$ and $b, c \in \mathbb{Z}$.
The graph of $f$ cuts the $x$ axis at the points where $x = -3$ and $x = 2$.

(i) Find the value of $b$ and the value of $c$.

(ii) Find the value of $x$ for which $f(x) = f(x + 2)$. 

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6. (a) Find the solution set of the inequality: \( 6 - 2x \leq 12, \ x \in \mathbb{R} \).

(b) Let \( f \) be the function \( f : x \rightarrow 5 - 3x - 2x^2 \) and \( g \) be the function \( g : x \rightarrow -2x - 1 \).

Using the same axes and scales, draw the graph of \( f \) and the graph of \( g \), for \(-3 \leq x \leq 2, \ x \in \mathbb{R} \).

(c) Use your graphs from part (b) to estimate:

(i) the maximum value of \( f(x) \)

(ii) the values of \( x \) for which \( f(x) = g(x) \)

(iii) the range of values of \( x \) for which \( f(x) \geq g(x) \).