Coimisiún na Scrúduithe Stáit
State Examinations Commission

JUNIOR CERTIFICATE EXAMINATION, 2003

MATHEMATICS – HIGHER LEVEL

PAPER 1 (300 marks)

THURSDAY, 5 JUNE – MORNING, 9:30 to 12:00

Attempt **ALL** questions.

Each question carries 50 marks.

Graph paper may be obtained from the superintendent.

The symbol ✍ indicates that supporting work must be shown to obtain full marks.

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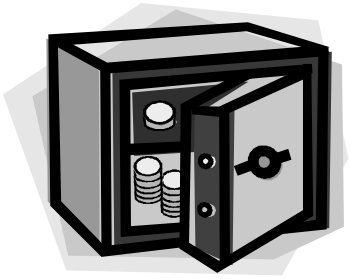
1. **(a)** ✍ Express 45 centimetres as a fraction of 15 metres and write your answer in its simplest form.

(b) €6000 was invested at compound interest.
The rate for the first year was 4% per annum.



(i) ✍ Calculate the amount of the investment at the end of the first year.

(ii) ✍ At the end of the second year the investment amounted to €6520·80. Calculate the rate per annum for the second year.

(c) The standard rate of income tax is 20% and the higher rate is 42%.
Fiona has tax credits of €1493 for the year and a standard rate cut-off point of €30 000.
She has a gross income of €31 650 for the year.

(i) ✍ After tax is paid, what is Fiona's income for the year?

(ii) ✍ What would Fiona's gross income for the year need to be in order for her to have an after-tax income of €29 379?

2. **(a)** **(i)** List the first six multiples of 3 and the first six multiples of 5.

(ii) Hence, write down the lowest common multiple of 3 and 5.

(b) **(i)** ✍ By rounding to the nearest whole number, estimate the value of

$$\frac{1}{3\cdot 67} + (7\cdot 9)^2 \times \sqrt{16\cdot 32}\,.$$

Then, evaluate $\dfrac{1}{3\cdot 67} + (7\cdot 9)^2 \times \sqrt{16\cdot 32}$, correct to two decimal places.

(ii) ✍ Simplify $\dfrac{\sqrt[3]{27} \times 3}{9^{\frac{1}{2}} \times 3^4}$ into the form 3^n where $n \in \mathbf{Z}$.

(c) **(i)** $A = \{1, 2, 3, 4\}$, $B = \{2, 3, 5\}$ and $C = \{1, 3, 4, 5, 6\}$.

✍ List the elements of $(A \setminus B) \cup (C \cap B)$ and the elements of $(A \cup B) \cap (C \setminus B)$.

(ii) U is the universal set and P and Q are two subsets of U.

$\#U = 20$
$\#(P \cap Q) = x$
$\#(P \setminus Q) = 2x$
$\#((P \cup Q)') = 4$
$\#Q = 2(\#P)$.

✍ Represent the above information on a Venn diagram and hence find $\#Q$.

3. **(a)** ✍ Given that $p = \frac{x + 2y}{3}$, express y in terms of x and p.

(b) **(i)** ✍ Multiply out: $(3x - 1)(2x^2 + x - 4)$.

(ii) ✍ Evaluate your answer to part **(i)** when $x = -2$.

(c) **(i)** ✍ Solve $x^2 - 13x + 36 = 0$.

(ii) ✍ Hence, find the two values of $t \in \mathbf{R}$ for which

$$\left(\frac{1}{t} + 2\right)^2 - 13\left(\frac{1}{t} + 2\right) + 36 = 0.$$

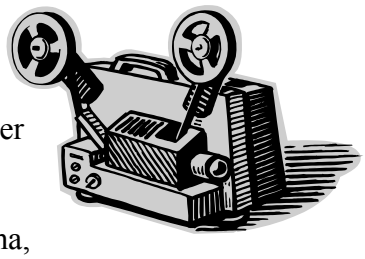
4. **(a)** ✍ List the solution set of the inequality

$$-3x - 3 > x - 12, \quad x \in \mathbf{N}.$$

(b) **(i)** Factorise $4x^2 - 49$.

(ii) ✍ Factorise $ab - cb + ac - c^2$.

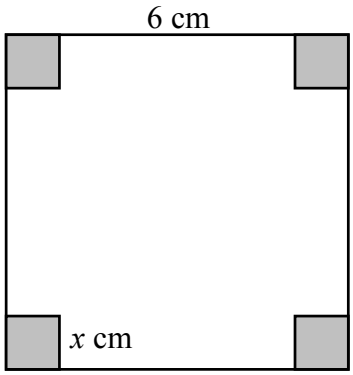
(c) A cinema takes in €400 each time that all seats are sold.
Next week, eight seats will be removed to make room for a new emergency exit.
The price per seat will have to be increased by €2·50 in order to keep the takings at €400.

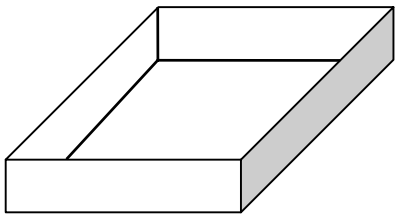


(i) Taking x to be the number of seats now in the cinema, write an equation in x to represent the above information.

(ii) ✍ Solve the equation to find the number of seats now in the cinema and the price per seat now.

5. A square sheet of cardboard measures 6 cm by 6 cm.
A square of side x cm is removed from each corner.
The remaining piece of cardboard is folded to form an open box as shown.





(a) ✍ Show that the area, in cm^2, of each side of the box is $6x - 2x^2$.

(b) ✍ Let f be the function $f : x \to 6x - 2x^2$.

Evaluate $f(x)$ when $x = 0, 1, 2, 3, 4$.

Hence, draw the graph of f for $0 \le x \le 4, \quad x \in \mathbf{R}$.

(c) Use your graph from part **(b)** to estimate:

(i) ✍ the area of a side when $x = 0{\cdot}5$

(ii) ✍ the maximum possible area of a side

(iii) ✍ the value of x that gives sides of maximum area

(iv) the length and height of a side of maximum area.

6. **(a)** ✍ Solve $3(x-2) + 1 = 19$ and verify your answer.

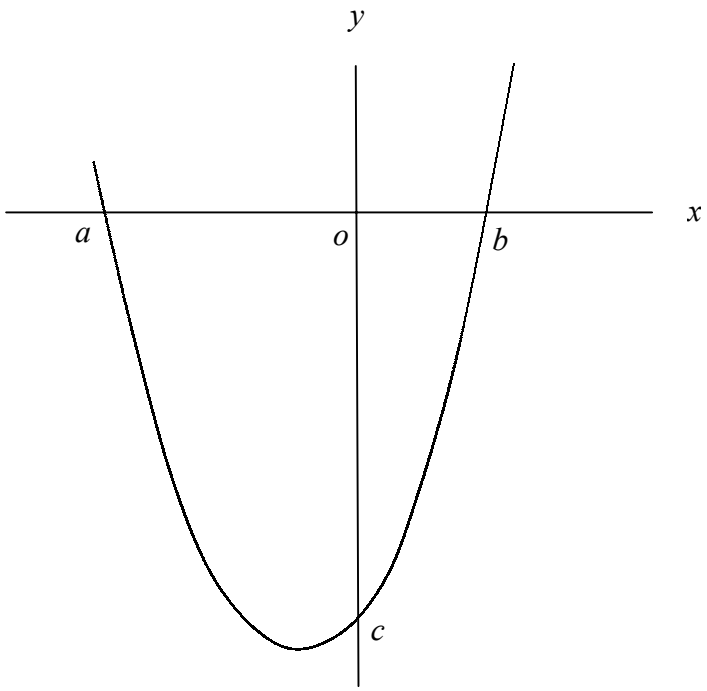
(b) **(i)** ✍ Solve the simultaneous equations:

$$3x + 4y = -1$$

$$2x + 9 = -6y.$$

(ii) ✍ By graphing the two lines on a single co-ordinate diagram, check your answer to part **(i)**.

(c) The diagram shows part of the graph of the function $f : x \to x^2 + 2x - 8, \quad x \in \mathbf{R}$.



(i) ✍ The graph intersects the x-axis at a and b and the y-axis at c.
Find the co-ordinates of a, b, and c.

(ii) Hence, write down the range of values of x for which $x^2 + 2x - 8 \leq 0$.