

AN ROINN OIDEACHAIS AGUS EOLAÍOCHTA**JUNIOR CERTIFICATE EXAMINATION, 2002****MATHEMATICS - HIGHER LEVEL****THURSDAY, 6 JUNE - MORNING, 9.30 to 12.00****PAPER 1 (300 marks)**

Attempt **QUESTION 1** (100 marks) and **FOUR** other questions (50 marks each).

**WARNING: Marks will be lost if necessary work is not clearly shown.
Mathematics Tables may be obtained from the Superintendent.**

1. (i) An estate agent's fee for selling a house is €1350.
This fee is 3% of the selling price of the house. Calculate the selling price.
- (ii) A person travelled at an average speed of 72 km/hr for 4 hours and 20 minutes.
How far did the person travel?
- (iii) A box is in the shape of a cube of side 7 cm.
Find the volume of the largest sphere which will fit exactly in the box.
Take $\pi = \frac{22}{7}$.
- (iv) Evaluate $\sqrt{\frac{1}{0.25}} + (0.6)^2$.
- (v) If $\frac{3}{a} = \frac{4}{b} - \frac{1}{c}$, express c in terms of a and b .
- (vi) Find the value of n for which $\frac{4}{2^{n+1}} = 32$.

(vii) If $\log_3 p = 5$, calculate the value of p .

(viii) If $x * y = x^2 + 2y + 3$, find the two values of a for which $a * a = 6$.

(ix) Express $\sqrt{72} - \sqrt{8}$ in the form $k\sqrt{2}$ where $k \in \mathbf{N}$.

(x) Solve the equation $x^2 - x - 6 = 0$.

Hence, or otherwise, solve the inequality $x^2 - x - 6 \leq 0$, $x \in \mathbf{R}$.

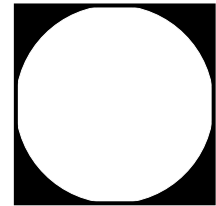
2. (a) A circle fits exactly inside a square of area 49 cm^2 as shown.

Calculate

(i) the length of a side of the square

(ii) the area of the shaded region.

Take $\pi = \frac{22}{7}$.



(b) A cone and a sphere both have radius 2 cm.

The curved surface area of the cone equals the surface area of the sphere.

Find the slant height of the cone.

(c) Water flows through a cylindrical pipe of internal diameter 1 cm at a speed of 2 cm per second.

(i) Verify that the rate of flow is $\frac{11}{7} \text{ cm}^3$ per second, taking $\pi = \frac{22}{7}$.

(ii) The water from the pipe flows into an empty hemispherical bowl.

It takes 36 seconds to fill the bowl. Calculate the internal radius of the bowl.

3. (a) Factorise fully each of the following:

(i) $x^2 - 7x + 12$

(ii) $4x^2 - 25y^2$

(iii) $27x^3 + y^3$.

(b) Simplify

$$(2x^3 + 5x^2 - 14x + 3) \div (2x - 3).$$

(c) (i) Solve, correct to one decimal place, the equation

$$x^2 - 4x + 2 = 0.$$

(ii) Use your answers to part (i) to find, correct to one decimal place, the two values of k for which

$$(k - 5)^2 - 4(k - 5) + 2 = 0.$$

4. The amounts of money spent by 100 customers in a shop are recorded in the following grouped frequency table:

Amount spent (€)	0 - 10	10 - 20	20 - 30	30 - 40	40 - 50
Number of customers	22	30	21	15	12

(Note: 0 - 10 means €0 or more, but less than €10, etc.)

(i) Draw a histogram to illustrate this information.

(ii) Use the mid-interval values to calculate the mean amount of money spent per customer.

(iii) Copy and complete the cumulative frequency table:

Amount spent (€)	< 10	< 20	< 30	< 40	< 50
Number of customers		52			

(iv) On graph paper, draw the ogive (cumulative frequency curve).

(v) Use your graph to estimate the number of customers who spent the median amount or more, but less than the mean amount.

5. (a) Using the same axes and scales, draw the graphs of

$$f : x \rightarrow 2x^2 - 2x - 3$$

$$g : x \rightarrow 2 - 3x$$

in the domain $-2 \leq x \leq 3$, $x \in \mathbf{R}$.

Use your graphs to estimate

- (i) the minimum value of $f(x)$
- (ii) the values of x for which $f(x) = g(x)$.

- (b) $h : x \rightarrow 3x + p$ and $k : x \rightarrow 4x^2 - p$ are two functions defined on \mathbf{R} , where $p \in \mathbf{Z}$.

- (i) If $h(2) = 4$, find the value of p .
- (ii) Hence, find $(h \circ k)(-1)$.
- (iii) Find the two values of x for which $h(x) + k(x) = 0$.

6. (a) $A = \{1, 2, 3, 4, 5\}$, $B = \{4, 5, 6, 7\}$, $C = \{3, 4, 7, 8\}$.

List the elements of:

- (i) $A \Delta B$
- (ii) $(A \setminus B) \Delta C$.

- (b) Solve the simultaneous equations:

$$2x - y = 5$$

$$x + 3y = \frac{x-4}{2}.$$

- (c) A prize fund of €1000 was shared equally between x people.

If there had been one person less, each person would have received €50 more.

Write an equation in x to represent this information.

Solve this equation for x and verify your answer.