

JUNIOR CERTIFICATE EXAMINATION, 1992

MATHEMATICS – HIGHER LEVEL – PAPER 1 (300 marks)

THURSDAY, 11 JUNE – MORNING, 9.30 to 12.00

Attempt **QUESTION 1** (100 marks) and **FOUR** other questions (50 marks each).

Marks may be lost if all your work is not clearly shown.

Mathematics Tables may be obtained from the Superintendent.

1. (i) A solicitor's fee for the sale of a house is $1\frac{1}{2}\%$ of the selling price.
If the fee is IR£870, calculate the selling price.

- (ii) Interest of 6% per annum is paid on an investment of IR£3500.

Tax at the rate of 30p in the IR£ is paid on the interest.

How much is the investment worth at the end of the first year?

- (iii) Evaluate

$$\sqrt{\frac{4}{1 - (0.6)^2}}$$

- (iv) If $r = \frac{1}{p} + \frac{1}{q}$, express p in terms of r and q .

- (v) If $f: x \rightarrow 3x - 2$ and $g: x \rightarrow 2x - 3$,
find the value of x for which $(f \circ g)(x) = 1$.

- (vi) The mean of 15 numbers is 4.

The mean of 10 of these numbers is 2.

What is the mean of the remaining 5 numbers?

- ~~(vii)~~ If $9^x = 3$ and $\log_y x = \frac{1}{2}$, find the value of y . ~~*~~

- (viii) If x_1 and x_2 are the roots of the equation

$$3x^2 + kx + 1 = 0$$

and $x_1 + x_2 = 2$, find k .

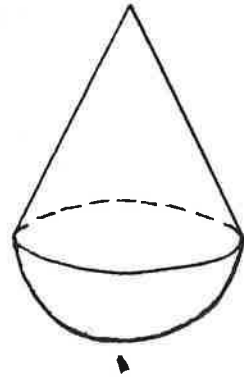
- (ix) If $a * b = 3ab - (a + b)$, find the two values of a for which $a * a = 1$.

- (x) Graph on the number line the solution set of

$$4 - 3x - x^2 \leq 0, \quad x \in \mathbf{R}.$$

2.

- (a) A toy is made of a cone which fits exactly on top of a hemisphere, as shown in the diagram. The radius length of the hemisphere is 6 cm and the volume of the cone is half the volume of the hemisphere. Calculate the total height of the toy.



- (b) The internal height of a cylinder is 20 cm. 50 identical solid metal cubes, each of edge 2 cm, are placed in the cylinder. Water is then poured into the cylinder until it is full.

If the volume of water needed is 1.17 litres, calculate the radius length of the cylinder.

Take $\pi = 3.14$.

3.

- (a) Factorise

(i) $3x^2 - 34x - 24$

(ii) $4x^2 - (3x - 2y)^2$

- (b) If

$$a(x^2 - 2x + 1) + b(2x^2 - 6x + 4) = x^2 + x - 2$$

for all values of x , write down two equations in a and b and then find the value of a and the value of b .

- (c) Solve for x :

$$\frac{x - 2}{x + 1} - \frac{x + 1}{x - 2} = 1$$

and give your answers correct to the nearest integer.

4.

The range of percentage marks obtained by 90 pupils is indicated in the following cumulative frequency table:

MARK	<10%	<30%	<40%	<60%	<90%	<100%
NUMBER OF PUPILS	6	21	45	80	90	y

- (i) Write down the value of y and say how many pupils were awarded a mark over 90%.
- (ii) Draw the ogive for this distribution, writing pupil numbers on the vertical axis.
- (iii) Estimate the median of the distribution.
- (iv) Estimate the interquartile range.

5. (a) Write down the values of $2n$ for $n = 1, 2, 3, 4, 5$.

Write down the values of $2n + 1$ for $n = 1, 2, 3, 4, 5$.

If $n \in \mathbb{N}$, say which of the following is an even integer and which is an odd integer:

(i) $(2n + 1)^2$

(ii) $(2n + 1) + (2n + 1)$

- (b) Using the same axes and the same scales, graph the two functions:

$$f: x \rightarrow 10 + x - 2x^2, \quad -2 \leq x \leq 3, \quad x \in \mathbb{R}.$$

$$g: x \rightarrow 3x - x^2, \quad 0 \leq x \leq 3, \quad x \in \mathbb{R}.$$

$f(x)$ is the height in km reached by an incoming missile launched at 5 p.m. ($x = -2$).

$g(x)$ is the height in km reached by an intercepting missile launched from the ground at 5.10 p.m. ($x = 0$).

Use your graphs to estimate

- (i) the maximum height reached by the incoming missile.
- (ii) the height at which the two missiles meet
- (iii) the time at which the two missiles meet.

6. (a) Solve for x and y :

$$3x + 5y = 13$$

$$\frac{2x - 1}{3} - \frac{4y - 5}{5} = -6$$

- (b) A person rowed a loaded boat across a lake, a distance of 5 km, and rowed the return journey empty. The total journey took 3 hours.

The boat travelled at a speed of $(x - 4)$ km/h on the outward journey and at a speed of $(x + 4)$ km/h on the return.

Calculate x .