

INTERMEDIATE CERTIFICATE EXAMINATION, 1972

MATHEMATICS—HIGHER COURSE—PAPER II
(300 marks)

TUESDAY, 13th JUNE—MORNING, 9.30 to 12

Six questions to be attempted.

All questions are of equal value.

Mathematical tables may be obtained from the Superintendent.

1. Find the compound interest on £800 when invested for two years at 8% per annum. What sum of money invested at 8% per annum compound interest would give £56.16 as the interest for the second year?

2. (a) Solve the simultaneous equations

$$y = 2x - 5$$

$$x = 3y + 10.$$

- (b) Factorise $3x^2 - 3x - 6$ and hence, or otherwise, find the solution set of $3x^2 - 3x - 6 \geq 0$ and graph that set on the numberline.

3. (a) If $S = \{a, b, c\}$ and $R = \{1, 2, 3\}$, write down

(i) the couples of $S \times R$

(ii) a subset of $S \times R$ that is a relation and not a function

(iii) a subset of $S \times R$ that is a bijection.

(b) If $E = \{p, q\}$, write down the elements of $\mathcal{P}E$ (i.e. the set of subsets of E) and graph the relation \subset defined on $\mathcal{P}E$.

Is \subset an order relation? Explain your answer.

4. (a) Let $A = \{1, 2, 3, d\}$ and $B = \{1, d\}$. Write down the elements of $A \cap B$ and of $A \cup B$. If X is a set such that $B \cup X = A$, write down the elements of each possible X .

- (b) Use the diagram to illustrate on your answer book the set

$$A \setminus (B \cup C).$$

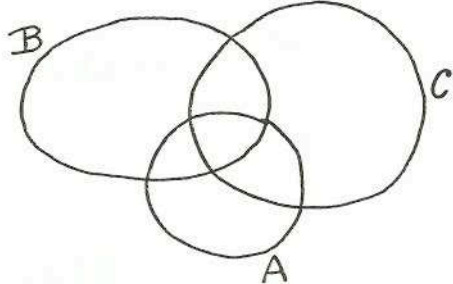
On separate diagrams illustrate

(i) $(A \setminus B)$ (ii) $(A \setminus C)$.

Use your diagrams to verify the truth or otherwise of the two statements

$$A \setminus (B \cup C) = (A \setminus B) \cup (A \setminus C)$$

$$A \setminus (B \cap C) = (A \setminus B) \cap (A \setminus C).$$



5. A prize of £240 was shared equally by a certain number of prizewinners. If the number of prizewinners had been less by 6, each share would have been £2 greater. How many prizewinners were there?

6. (a) The n th term of a sequence is

$$\frac{n+1}{n^2+n+1}.$$

Write down the first four terms and say, giving a reason, whether the sequence is arithmetic or not.

The n th term of an arithmetic sequence is $3n - 1$; find the sum of the first 20 terms.

- (b) Explain why the limit of $\frac{a(1-r^n)}{1-r}$ is $\frac{a}{1-r}$ as n gets very large and $0 < r < 1$.

Express the recurring decimal $0.1\bar{5}$ ($= 0.1555\dots$) in the form $\frac{m}{n}$, where $m, n \in \mathbb{N}$, $n \neq 0$.

7. (a) Write the denary number 40.75 in binary form. Convert the binary number 110001.0101 to denary form.

(b) (i) If x and \sqrt{x} are rational numbers, write down the domain of x for which $x < \sqrt{x}$.

(ii) Find the value of each of the following:

$$(2\frac{1}{4})^{1/2},$$

$$\log_5 125,$$

$$\log_{25} 5.$$

(iii) If $x = \log_3 2$, show that $\log_3 (2\frac{1}{4}) = 2(1-x)$.

[P.T.O.]

8. Graph the function $f : x \rightarrow 2(x-1)^2 (= y)$ for values of x in the domain $-1 \leq x \leq 3$. Show that the line $x = 1$ is an axis of symmetry of the graph and find the minimum of $f(x)$ and the value of x that gives this minimum.

Verify from your graph that there is no solution of the equation

$$2(x-1)^2 = -x,$$

where x is a rational number.

9. A market gardener recorded in a cumulative frequency table the produce from his tomato plants:

Number of Tomatoes per plant = n	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Number of plants producing n tomatoes at most	2	2	5	13	20	27	38	46	58	64	69	75	76	77	79	80

- Draw a graph to illustrate the data. Find
- the total number of tomato plants,
 - how many plants produced 4 tomatoes per plant?
 - what percentage of plants yielded 10 or more tomatoes per plant?
 - the median number of tomatoes.
10. Indicate the set of points, S , given that the coordinates of each point in S simultaneously satisfy the inequalities

$$1 < x < 4; \quad 2y > 1; \quad x - y + 3 > 0.$$

If (x, y) are the coordinates of a point in S , find

- the maximum value of $x + 2y$
- the minimum value of $x + 2y$.

