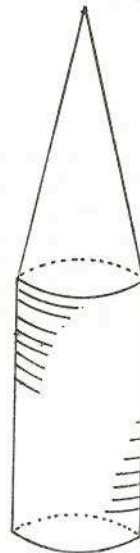


INTERMEDIATE CERTIFICATE EXAMINATION, 1970

MATHEMATICS — HIGHER COURSE — PAPER I
(300 marks)

WEDNESDAY, 10th JUNE — MORNING 9.45 to 12.15

Six questions to be answered.
All questions are of equal value.
Mathematical Tables may be had from the Superintendent.



1. A vessel in the shape of a cone standing on a cylinder, as in diagram, has water in it to a depth of 2 cm. and the internal diameter of the base of the cylinder is 4 cm. in length.
When turned upside down the water occupies half the capacity (internal volume) of the cone. Calculate the height of the cone.

2. (a) Express 30 miles per gallon in kilometres per litre. Give your answer correct to the nearest integer.

(take 1 mile = 1.609 kilometres
1 gallon = 4.546 litres)

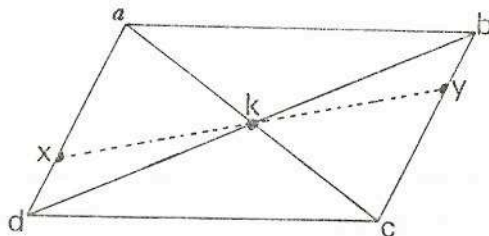
- (b) Two cubical boxes have the lengths of their edges in the ratio 2 : 3.

Find (i) the ratio of their surface areas,
(ii) the ratio of their volumes.

3. Prove that the diagonals of a parallelogram bisect each other.

$abcd$ is a parallelogram and the diagonals intersect in k . By considering S_k , the central symmetry in k or otherwise, prove that the diagonal $[ac]$ bisects the area of $abcd$.

x and y are points in $[ad]$ and $[bc]$, respectively, such that $k \in [xy]$. Prove that k bisects $[xy]$ and hence deduce that $aycx$ is a parallelogram.

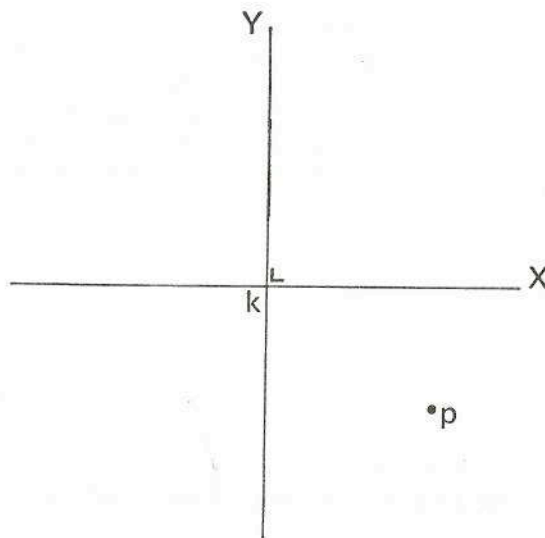


4. Explain what is meant by the statement:

“Equipollence is a transitive relation.”

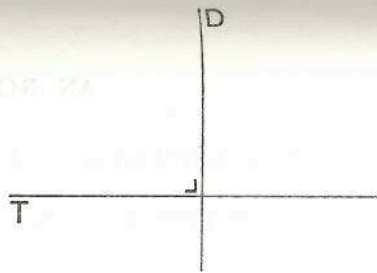
Illustrate your answer by means of a diagram.

If $(a, b) \uparrow (c, d)$ and $(c, d) \uparrow (e, f)$, show that ae is parallel to bf .



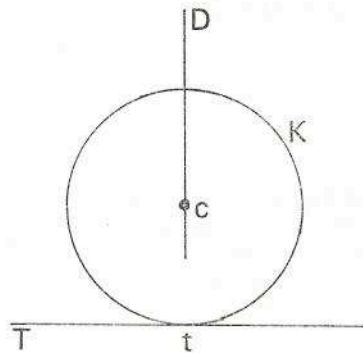
X and Y are two lines at right angles, as in diagram, and they intersect at k . p is a point as shown. Construct the image of p by the composite reflection $S_Y \circ S_X$ (i.e. reflect first in X and then in Y) and prove that $S_Y \circ S_X = S_k$, where S_k is the central symmetry in k .

5. In the diagram $D \perp T$.
What is the image of T by S_D , the reflection in D ?



Assuming that reflection conserves (preserves) length, prove that the image of a circle K by reflection in a line containing c , the centre of K , is K .

T is the tangent to the circle K at t , as shown in the diagram, and c is the centre of K . If $D \perp T$, find the image of $K \cap T$ by S_D (reflection in D) and deduce that the tangent to a circle is perpendicular to the diameter of the circle which contains the point of contact.



6. $[ab]$ and $[cd]$ are two chords of a circle which intersect internally at k .
Prove that $|ak| \cdot |kb| = |ck| \cdot |kd|$.

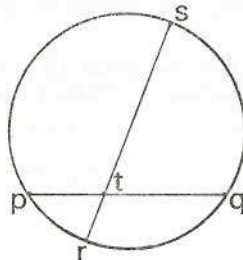
In the diagram $[sr]$ and $[pq]$ are two chords of a circle of radius 5 cm. If

$$|pt| = 3.36 \text{ cm.}$$

$$|tq| = 6 \text{ cm.}$$

$$|tr| = 2.8 \text{ cm.}$$

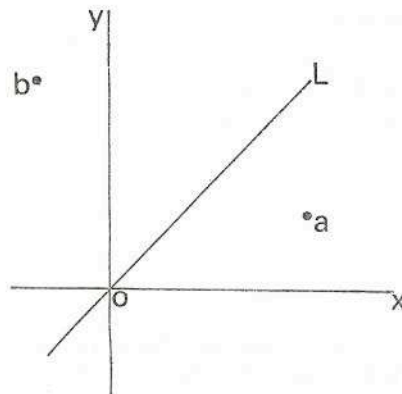
calculate the distance of t from the centre of the circle.



7. (a) Find the distance between the points $(-1, 7)$ and $(-4, 3)$.
(b) Show that the line determined by the points $(7, 13)$ and $(3, 5)$ is parallel to the line determined by the points $(\frac{1}{2}, 3\frac{1}{2})$, $(-2\frac{1}{2}, -2\frac{1}{2})$.

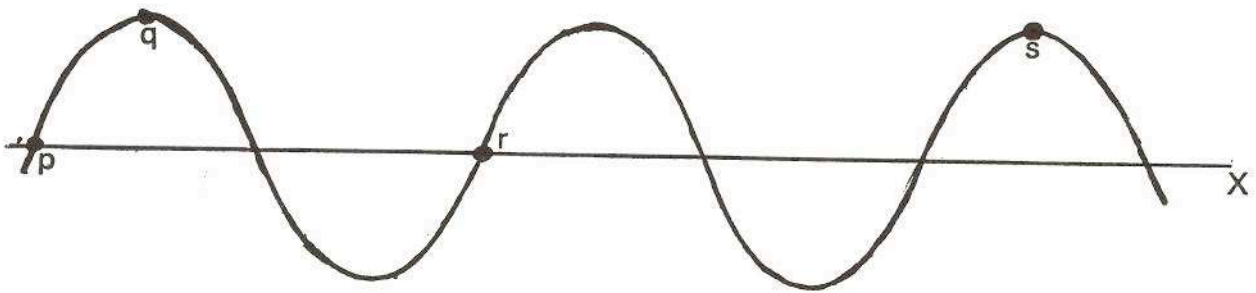
- (c) In the diagram, L is the line $y = x$ and the point b is the image of the point a by the composite reflection $S_Y \circ S_L$.

- (i) Show that $oa \perp ob$.
(ii) If $(3, 1)$ are the coordinates of a , find the coordinates of b .
(iii) Find the gradient (slope) of oa and the gradient of ob and calculate the product of these gradients.



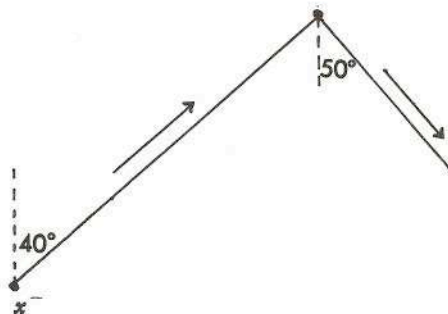
8. (a) Show that the line $y = x$ is perpendicular to the line $x + y = 4$.
Hence, or otherwise, show that the perpendicular distance of the point $(5, 5)$ from the line $x + y = 4$ is $3\sqrt{2}$.
(b) Find the equation of the image of the line $y = 2x$ by the translation $(0, 0) \rightarrow (1, 4)$.

9. (i) (a) Construct an angle A ($< \frac{\pi}{2}$) such that $\sin A = 0.7$.
 (b) If $\cos B = 0.4$, find, using your tables, the value of B . Write down one other value of B .
 (ii) Make two rough copies of the following graph in your answer book and then answer (c) and (d):



- (c) If the graph is that of $y = \sin x$, $x \in X$, fix a possible position for the Y -axis and hence write out the coordinates of the points p, q, r, s .
 (d) If the graph is that of $y = \cos x$, $x \in X$, fix a possible position for the Y -axis and hence write out the coordinates of the points p, q, r, s .

10. (i) A student starts out from x and walks 3 kilometres in a direction 40° East of North. He then walks in a direction 50° East of South until he is directly East of x . How far is he then from x ? Give your answer correct to one place of decimals. See diagram.



- (ii) a and b are two trees on the edge of a lake and c is a tree some distance from it.

$$|ca| = 50 \text{ metres}$$

$$\angle acb = 70^\circ$$

$$\angle cab = 85^\circ$$

Calculate $|ab|$ correct to three significant figures.

