AN ROINN OIDEACHAIS
INTERMEDIATE CERTIFICATE EXAMINATION, 1968
MATHEMATICS - GEOMETRY
MONDAY, 17TH JUNE - Morning, 10 to 12.30
Six questions to be answered.
Mathematical Tables may be had from the Superintendent.

1. If two sides of a triangle are equal, prove that the angles opposite those sides are also equal.
   State the converse theorem.
   In a triangle ABC there are 2\(\pi\) degrees in the angle ABC and \((90 - \alpha)\) degrees in the angle BAC
   (see diagram). Prove that the triangle ABC is isosceles.

2. (i) AB is a straight line and S is any point not in AB. Show, using ruler and compass, how to construct a straight line through S parallel to AB.
   (No proof required).
   (ii) Construct a triangle ABC in which AB = 3\(\frac{1}{2}\), \(\angle ACB = 120^\circ\), BD = 1\(\frac{1}{2}\) where
   D is the foot of the perpendicular from B to AC produced.
   (50 marks)

3. In a quadrilateral, if one pair of opposite sides are equal and parallel, prove that the other pair of opposite sides are also equal and parallel.
   ABCD is a parallelogram and E is any point on CD (between D and C). DC is produced to F
   so that CF = DE and EF is produced to M so that EM = AE. Prove that each of the
   quadrilaterals ABFE and BMPD is a parallelogram.
   (50 marks)

4. On a given line show how to construct segments of a circle (one on either side of the line) such that each segment contains an angle equal to a given angle.
   Hence show how to construct a triangle ABC, given BC, \(\angle BAC\) and such that A is on a given line.
   Is it always possible to construct such a triangle ? Draw separate diagrams to illustrate the cases where 0, 1, 2, 3, 4 such triangles can be drawn.
   (55 marks)

5. Prove that the perpendicular from the centre of a circle to a chord of the circle bisects the chord.
   AB is a radius of the circle ADC. Another
   circle drawn on AB as diameter has centre O
   (see diagram). AD is a straight line cutting
   the circles at P and D as shown in the diagram.
   Prove that
   (i) P is the mid-point of AD.
   (ii) \(\angle DOP = \angle BDC\).
   (55 marks)

6. (i) Without using the tables, construct and mark an angle \(A\) so that \(\tan A = \frac{1}{\sqrt{3}}\).
   (ii) In a triangle ABC, \(\tan \angle BAC = \frac{1}{\sqrt{3}}\), the perpendicular from B to AC is 2 inches in
   length and \(\sin \angle BCA = \frac{\sqrt{3}}{2}\). Calculate the area of the triangle.
   (35 marks)

7. When are two triangles said to be similar?
   If two triangles have one angle of one equal to an angle of the other and the sides about the equal angles
   are proportional, prove that the triangles are similar.
   ABC is a triangle inscribed in a circle as shown
   in diagram. AT is the tangent at A and \(\angle AEB = \angle ACT\).
   D and E are the mid-points of AB and AT, respectively.
   Prove
   (i) Triangles ABC, ATC are equiangular and hence
       that \(\angle AEB = \angle ACT\).
   (ii) Triangles DBC, EAC are similar.
   (iii) \(\frac{DC}{EC} = \frac{AB}{AT}\).
   (35 marks)