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INTERMEDIATE CERTIFICATE EXAMINATION, 1945.

MATHEMATICS.—GEOMETRY.

WEDNESDAY, 13th JUNE.—AFTERNOON, 3 to 5.30.

The total number of questions answered should not exceed six.
Mathematical Tables may be obtained from the Superintendent.

1. Prove that equal chords of a circle are equally distant from the centre.
What is the locus of the middle points of equal chords in a circle? [30 marks.]

2. Prove that the angle in a semi-circle is a right angle.
A is one of the points at which two circles cut one another. BC is a diameter of one circle, and AB, AC (produced if necessary) cut the other circle at L, M. Prove that LM is a diameter of this circle. [30 marks.]

3. Draw a triangle ABC. Construct and name the loci of:
   (i) the centres of circles that pass through B, C;
   (ii) the centres of circles that touch AB, AC;
   (iii) the centres of circles of radius 1 inch that touch BC. [30 marks.]

4. On a straight line BC, 3 inches long, construct a segment of a circle to contain an angle of 50°.
   Hence construct a triangle ABC, such that BC=3 inches, angle A=50°, and such that the perpendicular from A to BC may be 3 inches long.
   [No proof required but the construction lines should be clearly shown.] [30 marks.]

5. (i) Show how to construct a triangle equal in area to a given quadrilateral.
   (ii) Show how to construct an isosceles triangle on a base of given length equal in area to a given quadrilateral. [30 marks.]
6. Prove that in a triangle the square on the side subtending an acute angle is equal to the sum of the squares on the sides containing that angle diminished by twice the rectangle contained by one of those sides and the projection of the other side upon it.

Hence deduce the formula \( a^2 = b^2 + c^2 - 2bc\cos A \), where \( A \) is an acute angle in a triangle \( ABC \).

[30 marks.]

7. The external bisector of the angle \( A \) of a triangle \( ABC \) meets \( BC \) produced in \( E \); prove that \( AB : AC = BE : CE \).

If \( AB = 4" \), \( AC = 1\frac{1}{2}" \), \( BC = 3\frac{1}{2}" \), find the length of \( CE \).

[35 marks.]

8. \( P \) is a point on the diameter \( AB \) produced of a circle and \( PR \) is a tangent to the circle. The bisector \( PQ \) of the angle \( APR \) meets \( AR \) in \( Q \). Prove that the angle \( RQP = 45^\circ \).

[35 marks.]

9. \( ABC \) is a triangle in which \( B = 72^\circ 45' \), \( C = 51^\circ 13' \), and the length of the perpendicular from \( A \) to \( BC \) is 12.5 cm. Find the lengths of the three sides.

[35 marks.]

10. A pole stands vertically on level ground. The length of the shadow cast by the pole when the sun’s elevation is 40° is 7 feet longer than what it is when the sun’s elevation is 50°. Find the height of the pole.

[35 marks.]