Coimisiún na Scrúduithe Stáit
State Examinations Commission

Leaving Certificate Examination, 2022

Applied Mathematics – Ordinary Level

Friday, 24 June – Afternoon, 2:00 to 4:30

Five questions to be answered. All questions carry equal marks.

A *Formulae and Tables* booklet may be obtained from the Superintendent.

Take the value of g to be 10 m s^{-2}.

$\vec{\imath}$ and $\vec{\jmath}$ are unit perpendicular vectors in the horizontal and vertical directions, respectively, or eastwards and northwards, respectively, as appropriate to the question.

Marks may be lost if necessary work is not clearly shown.

Marks may be lost for omission of correct units with numerical answers.

Diagrams are generally not drawn to scale.

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1. A car starts from rest at point *P* and accelerates uniformly to a speed of 20 m s^{-1} in a time of 5 s. The car then decelerates to 15 m s^{-1} with a constant deceleration of 0.5 m s^{-2}. It continues at this speed for a further 8 s. The car then decelerates with a *different* constant deceleration until it comes to rest at point *Q*.

The total time for the journey from *P* to *Q* was 35 s.

(i) Sketch a speed-time graph of the motion of the car as it travels from *P* to *Q.*

(ii) Calculate the acceleration of the car during the first 5 s.

(iii) Calculate the distance travelled by the car during the first 5 s.

(iv) Calculate the distance travelled by the car while decelerating from 20 m s^{-1} to 15 m s^{-1}.

(v) Calculate the total distance between *P* and *Q*.

A motorcycle passes point *P* with a speed of *u* m s^{-1} at the same instant that the car starts from *P*. The motorcycle travels at speed *u* for 20 s before decelerating to rest so that it finishes at point *Q* at the same instant as the car.

(vi) Find the value of *u*.

2. At noon, ship A is 100 km west of ship B. Ship A has a speed of $3\sqrt{3}$ km h^{-1} in a direction east 30° south. Ship B has a speed of $4\sqrt{2}$ km h^{-1} in a direction west 45° south.

(i) Draw a diagram to show the positions and velocities of the two ships.

(ii) Write the velocity of ship A and the velocity of ship B in terms of $\vec{\imath}$ and $\vec{\jmath}$.

(iii) Calculate $\overrightarrow{v_{AB}}$, the velocity of ship A relative to ship B, in terms of $\vec{\imath}$ and $\vec{\jmath}$.

(iv) Calculate the magnitude of $\overrightarrow{v_{AB}}$ to one decimal place.

(v) Calculate the direction of $\overrightarrow{v_{AB}}$ to one decimal place.

(vi) Show that the shortest distance between the ships is 16.3 km.

(vii) Calculate the time when the ships are closest to each other.

3. **(a)** A particle is projected from a point *A* on a horizontal plane with an initial velocity of $25\vec{\imath} + 15\vec{\jmath}$ m s^{-1}. The particle lands at a point *B*, which is a distance *d* m from *A*.

Calculate

(i) the time of flight of the particle

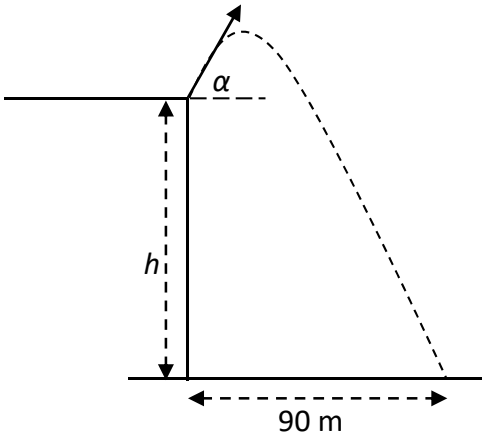
(ii) the value of *d*

(iii) the maximum height of the particle above the horizontal plane

(iv) the speed of the particle after 1.75 s.

Note: *Question 3, part (b) is on the next page.*

(b) A particle is projected from the edge of a vertical cliff of height h m. The particle is projected with a speed of 26 m s^{-1} at an angle α above the horizontal, as shown in the diagram. Tan $\alpha = \frac{5}{12}$. The particle lands 90 m from the base of the cliff.

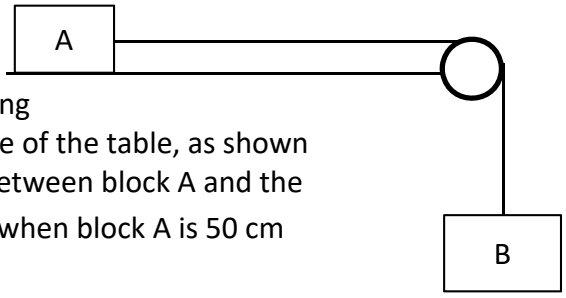


Calculate

(i) the time taken for the particle to land

(ii) the value of h.

4. **(a)** A block A, of mass 3 kg, is at rest on a rough horizontal table. It is connected to block B, of mass 7 kg, by a light inelastic string that passes over a smooth pulley at the edge of the table, as shown in the diagram. The coefficient of friction between block A and the table is $\frac{1}{3}$. The system is released from rest when block A is 50 cm from the edge of the table.



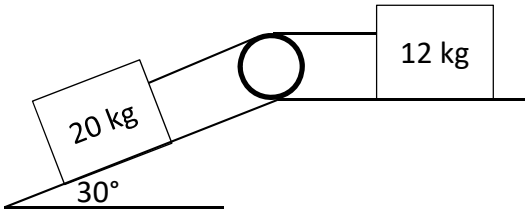
(i) Show on separate diagrams the forces acting on each block.

(ii) Show that the common acceleration of the blocks is 6 m s^{-2}.

(iii) Calculate the tension in the string.

(iv) Calculate the speed of block A when it reaches the edge of the table.

(b) Blocks of mass 20 kg and 12 kg are connected by a light inelastic string which passes over a smooth pulley, as shown in the diagram. The 20 kg block lies on a smooth surface which is inclined at 30° to the horizontal. The 12 kg block lies on a smooth horizontal surface.



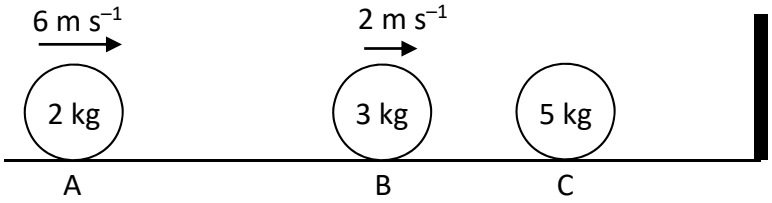
The system is released from rest.

(i) Calculate the acceleration of the system.

The surface beneath the 12 kg mass is replaced with another surface which has a coefficient of friction of μ.

(ii) Calculate the minimum value of μ so that the system does not move.

5. Three smooth spheres are positioned on a smooth horizontal table as shown. Sphere A, of mass 2 kg, moves towards sphere B with a velocity of 6 m s^{-1}. Sphere B, of mass 3 kg, has a velocity of 2 m s^{-1} in the same direction as sphere A. Sphere C, of mass 5 kg, is at rest. Sphere C is 12 m away from a wall.



The coefficient of restitution for the collision between sphere A and sphere B is $\frac{1}{4}$.

Calculate

(i) the speed of sphere A and the speed of sphere B after they collide

(ii) the loss of kinetic energy due to this collision

(iii) the magnitude of the impulse imparted to sphere A as a result of this collision.

Sphere B then collides with sphere C. Sphere B comes to rest as a result of this collision.

(iv) Calculate the value of the coefficient of restitution, *e,* between spheres B and C.

(v) Calculate the time C takes to reach the wall.

6. **(a)** Particles of weights 5 N, 2 N, 3 N and 10 N are placed at the points $(p, -1)$, $(-6, q)$, $(4, -5)$ and $(5, p)$ respectively.

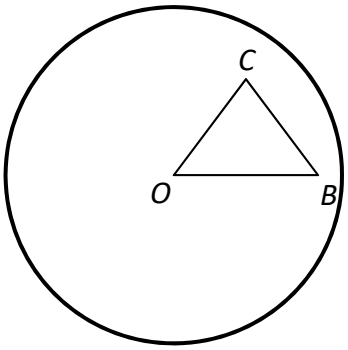
The co-ordinates of the centre of gravity of the system are $(3, q)$.

Calculate

(i) the value of p

(ii) the value of q.

(b) A uniform circular lamina with centre O and radius 7 cm has isosceles triangle OBC removed from it, where $|OB| = 6$ cm and $|OC| = |BC| = 5$ cm. Centre O and point B have co-ordinates (0, 0) and (6, 0) respectively.



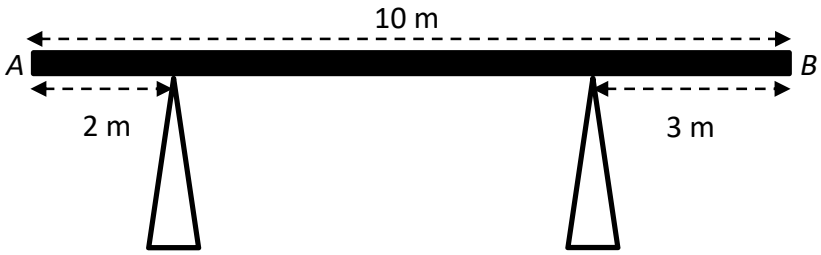
The remaining portion of the lamina has its centre of gravity at point P.

(i) Show that point C has co-ordinates (3, 4).

(ii) Calculate the co-ordinates of P to two decimal places.

(iii) Calculate $|OP|$ to two decimal places.

7. **(a)** A rod *AB*, of length 10 m and weight 800 N rests on two vertical supports, as shown in the diagram. The first support is placed 2 m from *A* and the second support is placed 3 m from *B*.



The rod is at rest and is in horizontal equilibrium.

(i) Draw a force diagram showing all the forces acting on the rod.

(ii) Calculate the normal reactions between the rod and each of the vertical supports.

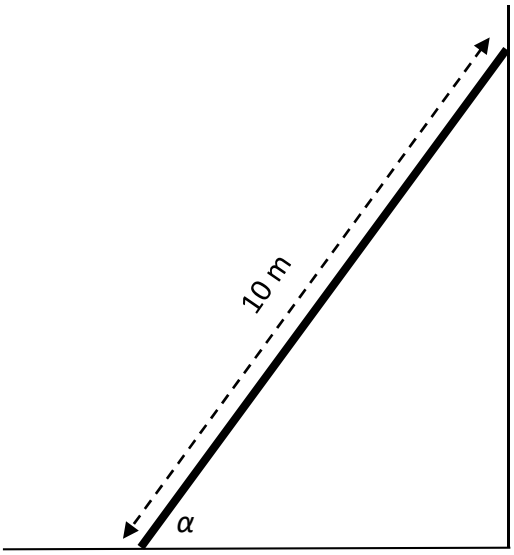
(b) A uniform ladder of length 10 m and weight 120 N rests on rough horizontal ground and leans against a smooth vertical wall.

The ladder is on the point of slipping when it makes an angle α with the ground, where $\tan \alpha = \frac{4}{3}$.

(i) Draw a force diagram showing all the forces acting on the ladder.

(ii) Calculate the normal reaction between the ladder and the wall.

(iii) Find the coefficient of friction between the ladder and the ground.



8. **(a)** A particle of weight 40 N moves with uniform circular motion in a horizontal circle of radius 2 m. It completes 50 rotations in one minute.

Calculate

(i) the mass of the particle

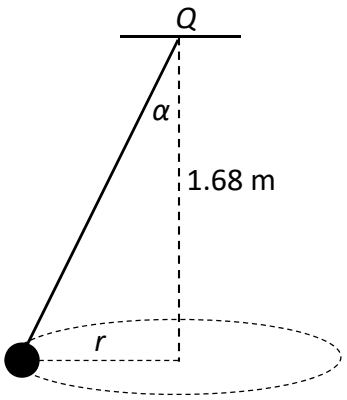
(ii) how many seconds the particle takes to complete one rotation

(iii) the angular velocity of the particle

(iv) the linear velocity of the particle

(v) the centripetal force acting on the particle.

(b) A particle of mass 3 kg is connected to a fixed point Q by a light inelastic string. The particle describes a horizontal circle of radius r m. The centre of the circle is 1.68 m vertically below Q, as shown in the diagram.



The string makes an angle α with the vertical, where $\tan \alpha = \frac{7}{24}$.

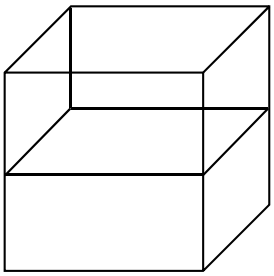
(i) Draw a force diagram showing all the forces acting on the particle.

(ii) Calculate r.

(iii) Calculate the tension in the string.

(iv) Calculate the speed of the particle.

9. **(a)** A cubic tank of side 50 cm and mass 8 kg is designed to hold a liquid of density 1200 kg m^{-3}.



The tank is *half-filled* with this liquid.

Calculate

(i) the total weight of the tank plus liquid

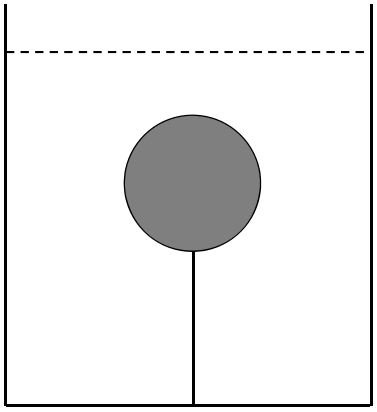
(ii) the pressure the liquid exerts on the bottom of the tank.

Another three litres of the liquid are added to the tank.

(iii) Calculate the total pressure the liquid now exerts on the bottom of the tank.

***Note**: 1 litre = 0.001 m^3*

(b) A solid sphere of radius *r* cm and density 300 kg m^{-3} is completely immersed in a tank containing a liquid of density 1500 kg m^{-3}.



The sphere is attached to the bottom of the tank by a string which has a tension of 11 N.

(i) Draw a force diagram showing all the forces acting on the sphere.

(ii) Calculate *r*.

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