Coimisiún na Scrúduithe Stáit
State Examinations Commission

Leaving Certificate Examination, 2021

Applied Mathematics – Higher Level

Friday, 25 June – Afternoon, 2:00 to 4:30

Five questions to be answered. All questions carry equal marks.

A *Formulae and Tables* booklet may be obtained from the Superintendent.

Take the value of g to be 9.8 m s^{-2}.

Marks may be lost if necessary work is not clearly shown.

Marks may be lost for omission of correct units with numerical answers.

Diagrams are generally not drawn to scale.

1. **(a)** A ball is thrown vertically downwards from the top of a building of height h m. The ball passes the top half of the building in 1.2 s and takes a further 0.8 s to reach the bottom of the building.
Find

(i) the value of h

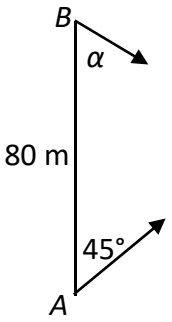
(ii) the speed of the ball at the bottom of the building.

(b) Car C, moving with uniform acceleration f passes a point P with speed u (> 0). Two seconds later car D, moving in the same direction with uniform acceleration $2f$ passes P with speed $\frac{6}{5}u$. C and D pass a point Q together. The speeds of C and D at Q are 6.5 m s^{-1} and 9 m s^{-1} respectively.

(i) Show that C travels from P to Q in $(\frac{3}{2f} + 5)$ seconds.

(ii) Find the value of f.

2. **(a)** A point B is 80 m north of a point A on a horizontal field. Alan is at point A of the field and Brian is at point B of the field. Alan starts to run in a straight line in the direction north 45° east at a constant speed of 2.5 m s^{-1}.



Brian sees Alan start to run, waits 8 seconds, and then runs from B to intercept Alan. Brian runs in a straight line in the direction south α° east at a constant speed of 4 m s^{-1} and intercepts Alan after t seconds.

Find

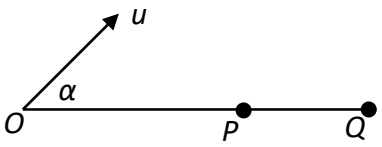
(i) the value of t

(ii) the value of α.

(b) Three aircraft, P, Q and R, are flying at the same height.
P is travelling north at 450 km h^{-1}. Q is travelling at $400\sqrt{2}$ km h^{-1} in a direction east 45° north.
R appears to the pilot of P to be flying in a direction east 10° south.
R appears to the pilot of Q to be flying in a direction east 15.67° south.

Find the magnitude and direction of the velocity of R.

3. **(a)** A particle is projected from a point O with speed u m s^{-1} at an angle α to the horizontal.



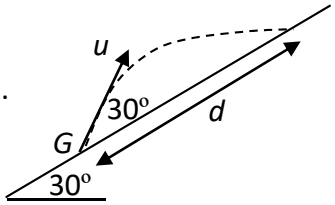
(i) Show that the range of the particle is $\frac{u^2 \sin 2\alpha}{g}$,

and that the maximum range $|OQ|$ is $\frac{u^2}{g}$.

If the angle of projection is increased to 60° the particle strikes the horizontal plane at P.

(ii) Find the distance $|PQ|$ in terms of u.

(b) A plane is inclined at an angle of 30° to the horizontal.
A particle is projected from a point G up the plane with initial speed u m s^{-1} at an angle of 30° to the inclined plane.
The range along the inclined plane is d.
The plane of projection is vertical and contains the line of greatest slope.

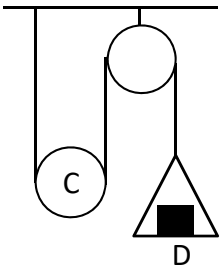


(i) Find d in terms of u.

When the particle strikes the inclined plane, it bounces vertically upwards.
The coefficient of restitution between the particle and the inclined plane is e.

(ii) Find the value of e.

4. **(a)** The diagram shows a light inextensible string having one end fixed, passing under a smooth movable pulley C of mass km kg and then over a fixed smooth pulley. The other end of the string is attached to a light scale pan. A bock D of mass m kg is placed symmetrically on the centre of the scale pan.
The system is released from rest. The scale pan moves upwards.

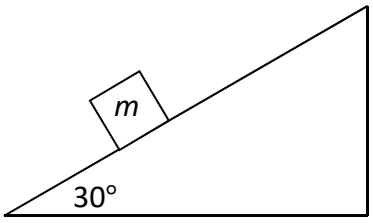


(i) Show that $k > 2$.

(ii) Find, in terms of k and m, the tension in the string.

(iii) Find, in terms of k and m, the reaction between D and the scale pan.

(b) A smooth wedge of mass $4m$ and slope 30° rests on a smooth horizontal surface. A particle of mass m is placed on the smooth inclined face of the wedge and is released from rest. A horizontal force F is applied to the wedge to keep it from moving.



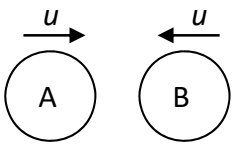
(i) Show, on separate diagrams, the forces acting on the wedge and on the particle.

(ii) Find F in terms of m.

If the force F is removed, the particle moves with acceleration p relative to the wedge and the wedge moves with acceleration q.

(iii) Find the value of p and the value of q.

5. **(a)** A smooth sphere A of mass $4m$, moving with speed u on a smooth horizontal table collides directly with a smooth sphere B of mass m, moving in the opposite direction with speed u.



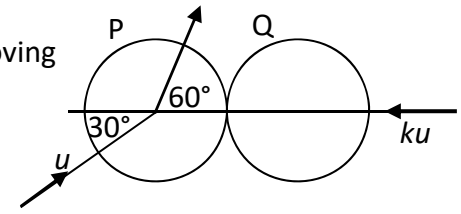
The coefficient of restitution between A and B is e.

(i) Find the speed, in terms of u and e, of each sphere after the collision.

The magnitude of the impulse on B due to the collision is T.

(ii) Show that $\frac{8mu}{5} \leq T \leq \frac{16mu}{5}$.

(b) A smooth sphere P has mass $2m$ and speed u. It collides obliquely with a smooth sphere Q of mass m which is moving with speed ku, as shown in the diagram.
Before the collision, the direction of P makes an angle of 30° to the line of centres. After the collision, the direction of P makes an angle of 60° to the line of centres.



The coefficient of restitution between the spheres is e.

(i) Show that $k = \frac{\sqrt{3}(1-e)}{2(1+e)}$.

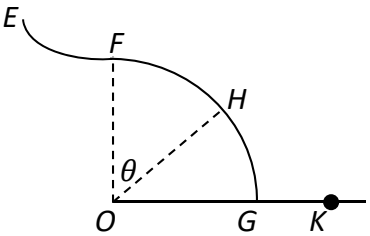
(ii) Find the speed of Q immediately after the collision.

6. **(a)** A particle D of mass m is suspended from a fixed point by a light elastic string of natural length ℓ and elastic constant $\frac{3mg}{\ell}$.
Initially D rests in equilibrium with the string vertical.
The particle is now pulled down a vertical distance $\frac{2}{3}\ell$ below its equilibrium position and released from rest.

(i) Show that D moves with simple harmonic motion.

(ii) In terms of ℓ, find the height above the equilibrium position to which D rises.

(b) A smooth slide EFG is in the shape of two arcs, EF and FG, each of radius r. The centre O of arc FG is vertically below F as shown in the diagram.
Point E is at a height $\frac{r}{5}$ above point F.
A child starts from rest at E, moves along the slide past the point F and loses contact with the slide at point H.
OH makes an angle θ with the vertical.

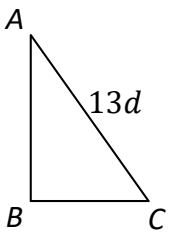


(i) Find the value of θ.

The child lands in a sandpit at point K.

(ii) Find, in terms of r, the speed of the child at K.

7. **(a)** A thin uniform rod, of length $30d$ and mass *m*, is bent to form a frame. The frame is in the shape of a right-angled triangle *ABC*, as shown in the diagram.
$|CA| = 13d$ and $|AB| > |BC|$.



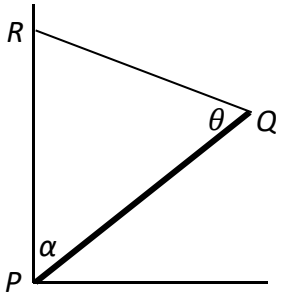
(i) Find $|BC|$ in terms of d.

(ii) Find the distance of the centre of gravity of the frame from *AB*.

The frame is freely suspended from *A*. A horizontal force of magnitude *kmg*, where *k* is a constant, is applied to the frame at *B*. The line of action of the force lies in the vertical plane containing the frame. The frame hangs in equilibrium with *AB* vertical.

(iii) Find the value of *k*.

(b) A uniform rod *PQ* of weight *W* rests against the junction of the horizontal ground and a vertical wall.
It is supported by a string of length ℓ attached to *Q* and to a point *R* vertically above *P* on the wall.
The rod makes an angle *α* with the wall and the string makes an angle θ with the rod, as shown in the diagram.
$|PR| = h$.

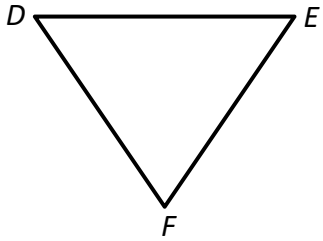


(i) Find T, the tension in the string, in terms of *W*, ℓ and h.

(ii) If $T = \frac{1}{3}W$, find ℓ in terms of h.

8. **(a)** Prove that the moment of inertia of a uniform rod, of mass *m* and length 2ℓ about an axis through its centre, perpendicular to its plane, is $\frac{1}{3}m\ell^2$.

(b) Three equal uniform rods, each of mass *m* and length 2ℓ, form the sides of a rigid equilateral triangular frame *DEF*.



The frame is free to rotate in a vertical plane about a fixed smooth horizontal axis which passes through *D* and is perpendicular to the plane of the frame.

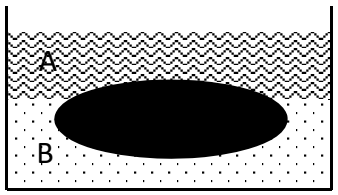
(i) Show that the moment of inertia of the frame about the axis is $6m\ell^2$.

The frame is held with *DE* horizontal and *F* below *DE*. It is then released from rest.

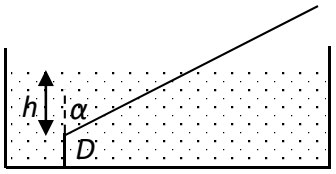
(ii) Find, in terms of ℓ, the angular speed of the frame when *FE* is horizontal for the first time.

(iii) If the period of small oscillations for the frame is 1.87 s, find the value of ℓ.

9. **(a)** Liquid A of relative density 0.8 rests on top of liquid B of relative density 1.4 without mixing. A solid object of relative density 1.2 floats with part of its volume in liquid A and the remainder in liquid B.
Find the fraction of the volume of the object immersed in liquid B.



(b) A uniform rod of length ℓ is tied by an inelastic string at its lower end *D* to the base of a tank. *D* is at a depth *h* below the surface of a liquid.
The relative density of the rod is *s*.
The relative density of the liquid is *ρ*.
The rod is in equilibrium, inclined at an angle *α* to the vertical as shown in the diagram.



(i) Show that the length of the immersed part of the rod is $\ell\sqrt{\frac{s}{\rho}}$.

If $4\rho h^2 = s\ell^2$ find

(ii) the value of *α*

(iii) *s* in terms of *ρ*, if the magnitude of the tension in the string is $\frac{1}{2}W$.

10. **(a)** A car of mass 1200 kg starts from rest and travels along a straight horizontal road. The engine of the car exerts a constant power of 3000 W.

If there is no resistance to the motion of the car, find

(i) the speed of the car after 3 minutes

(ii) the average speed of the car during this time.

(b) P, the population of insects in a region, grows at a rate that is proportional to the current population.

$$\frac{dP}{dt} = kP$$

where k is a positive constant. In the absence of any outside factors the population will triple in 15 days.

(i) Find the value of k.

A scientist begins to remove 10 insects from the population each day.

(ii) If there are initially 120 insects in the region the population will not survive. After how many days will the population die out?

There is no examination material on this page.

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