AN ROINN OIDEACHAIS

M.52

LEAVING CERTIFICATE EXAMINATION, 1991

APPLIED MATHEMATICS - HIGHER LEVEL

FRIDAY, 21 JUNE - MORNING, 9.30 - 12.00

Six questions to be answered. All questions carry equal marks.
Mathematics Tables may be obtained from the Superintendent.
Take the value of g to be 9.8 m/s^2.
Marks may be lost if all your work is not shown or you do not indicate where a calculator has been used.

1. (a) A particle starts from rest at a point p and accelerates at 2 m/s^2 until it reaches a speed v m/s.
It travels at this speed for 1 minute before decelerating at 1 m/s^2 to rest at q. The total time for the journey is 2 minutes.

    (i) Calculate the distance pq.
    (ii) If a second particle starts from p at time $t = 0$ and moves along pq with speed $(2t + 50)$ m/s, find the time taken to reach q.

   (b) A particle P is projected vertically upwards with an initial velocity u and two seconds later a second particle Q is projected vertically upwards from the same point with initial velocity $1.5u$. Calculate, in terms of u, how long Q is in motion before it collides with P and prove that $|u| > 9{\cdot}8$.

2. A runner observes that when her velocity is $u\vec{i}$ the wind appears to come from the direction $2\vec{i} + 3\vec{j}$, but when her velocity is $u\vec{j}$ the wind appears to come from the direction $-2\vec{i} + 3\vec{j}$.

    (i) Prove that the true velocity of the wind is

$$\frac{5u}{6}\vec{i} - \frac{u}{4}\vec{j}$$

    (ii) Find the speed and direction of motion of the runner when the wind velocity appears to be $u\vec{i}$.

3. A particle is projected, with speed u, down a plane which is inclined at an angle of 30° to the horizontal. The plane of projection is vertical and contains the line of greatest slope. The coefficient of restitution between the particle and the plane is e. The direction of projection makes an angle of 60° with the inclined plane.

    (i) Find the range to the first hop.
    (ii) For what value of e is the range for the second hop double the range for the first hop ?

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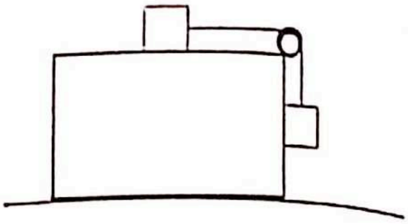
4. A rectangular block moves across a stationary horizontal surface with acceleration $g/3$. A particle of mass m, on the block, is connected by a string which passes over a light, smooth, fixed pulley to a second particle of mass m which presses against the block (see diagram).



(i) If contact between the particles and the block, is smooth, find the magnitude and direction of the resultant forces acting on the particles.

(ii) If contact between the particles and the block, is rough, for what same value of the coefficient of friction, will the particles remain at rest relative to the block ?

5. (a) A sphere of mass $4m$ travelling with speed u, strikes directly a stationary sphere of mass $2m$. If the coefficient of restitution is e, prove that the energy lost in the collision is

$$\frac{2mu^2}{3}(1 - e^2)$$

(b) A smooth sphere P, moving with velocity $4\vec{i} + 5\vec{j}$ m/s collides with an identical sphere Q moving with velocity $2\vec{i} + 3\vec{j}$ m/s where $\vec{i}$ is a unit vector along the line of centres at the moment of impact.
If e is the coefficient of restitution
(i) find the velocity of each sphere after impact.
(ii) calculate the value of e if P is deflected through an angle $\tan^{-1}(\frac{1}{4})$ as a result of the collision.

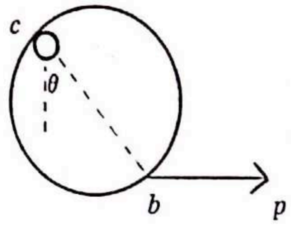
6. (a) A light elastic string of natural length 1.2 m is found to extend by 20 cm when a small mass is gently attached to one end, the other end being fixed. The mass is now made to describe a horizontal circle with angular velocity w rad/s.
Find an expression for the extension of the string in terms of w.

(b) A particle of mass 3 kg is suspended by a light string which is found to extend by 20 cm when the particle is at rest. If the upper end of the string is held firm and the particle is pulled down slightly and then released, find the period of the resultant motion.

7. A uniform circular hoop of weight W hangs over a rough horizontal peg c. A horizontal force p is applied at a point b where cb is a diameter of the hoop. When cb is inclined at an angle Θ to the downward vertical the system is in equilibrium and the hoop has not slipped.

(i) Find p in terms of W and Θ.

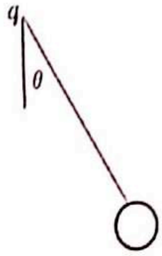
(ii) Show that the ratio of the frictional force to the normal reaction at the peg is $\dfrac{\tan\Theta}{2 + \tan^2\Theta}$



(iii) Show that, when the coefficient of friction is 1/2, the hoop never slips.

8. (a) Prove that the moment of inertia of a uniform rod of mass m and length $2l$, about an axis through its centre of mass perpendicular to the rod is $\frac{1}{3}ml^2$

(b) A uniform rod of mass m and length 88 cm has a uniform disc of mass m and radius 12 cm attached to one end. The rod and disc are in the same plane and the rod is collinear with a diameter of the disc (see diagram).



If the compound body is set in motion about an axis through q which is perpendicular to the plane of the rod and disc,

(i) find the period of small oscillations correct to two decimal places.

(ii) find the length of the equivalent simple pendulum.

9. State the Principle of Archimedes.
A uniform rod, of length p and relative density s is free to turn about its lower end, which is fixed at a depth h in a liquid of relative density q.

(i) Prove that the rod can float in an inclined position if

$$p^2 s > h^2 q$$

(ii) Calculate the value of s if the rod floats with three-quarters of its length immersed in water.

10. (a) Solve the differential equation

$$x\frac{dy}{dx} = \frac{1}{y} + y$$

if $y = 1$ when $x = 1$.

(b) A particle is projected in a straight line from a fixed point with velocity u at time $t = 0$. It is opposed by a resistance kv^n per unit mass. If s is the displacement at time t prove that when $v = 0$

$$s = \frac{u^{2-n}}{(2-n)k} \qquad \text{if } n < 1.$$