

## APPLIED MATHEMATICS – HIGHER LEVEL

FRIDAY, 23 JUNE – MORNING, 9.30 – 12.00

Six questions to be answered. All questions carry equal marks.  
 Mathematics Tables may be obtained from the Superintendent.  
 Take the value of  $g$  to be  $9.8 \text{ m/s}^2$ .  
 Marks may be lost if all your work is not shown or you do not indicate where a calculator has been used.

1. Two cars  $A$  and  $B$ , each 5 m in length, travel with constant velocity 20 m/s along a straight level road. The front of car  $A$  is 15 m directly behind the rear of car  $B$ . Immediately on reaching a point  $P$  each car decelerates at  $4 \text{ m/s}^2$ .
  - (i) Show that  $A$  collides with  $B$ .
  - (ii) At what distance from  $P$  does the collision occur ?
  - (iii) Show the motion of both cars on the same speed-time graph.
  
2. A man travelling due North at 20 m/s finds that the wind appears to blow from the West. When he travels due West at 8.45 m/s the wind appears to blow from the South West.
  - (i) Calculate the velocity of the wind.
  - (ii) If the man travelled in a direction  $30^\circ$  North of West at 8 m/s from what direction would the wind appear to blow ?
  
3. A particle is projected with speed  $u$  at an angle  $\alpha$  to the horizontal. The range of the particle on the horizontal plane through the point of projection is  $R$ .
  - (i) Show that  $R$  is a maximum when  $\alpha = 45^\circ$ .
  - (ii) If  $R = \frac{u^2}{2g}$ , find the two possible values of  $\alpha$ .
  - (iii) If the ratio of the greatest height to the range is 2 : 5, find  $\alpha$ .
  
4. State the laws governing the oblique collision of elastic spheres. A smooth sphere  $A$ , of mass  $m$ , moving with speed 0.6 m/s, impinges obliquely on a smooth sphere  $B$ , of mass  $2m$ , which is at rest. After the collision  $A$  is found to move with speed 0.2 m/s in a direction at right angles to its original direction.
  - (i) Find the direction of  $A$  before impact.
  - (ii) Find the coefficient of restitution.
  - (iii) Show that the loss of kinetic energy, as a result of the impact, is 0.06 m.
  
5. A wedge of mass 8 kg can slide freely on a smooth horizontal table. On one face, inclined at an angle of  $30^\circ$  to the horizontal, is placed a particle of mass 4 kg, and on the other face, inclined at an angle of  $60^\circ$  to the horizontal, is placed a particle of mass 6 kg. If both faces of the wedge are smooth,
  - (i) Show on separate diagrams the forces acting on each mass.
  - (ii) Show that when the particles are released from rest, the acceleration of the wedge is  $\frac{g}{9\sqrt{3}}$ .

6. Define Simple Harmonic Motion.

A mass of 4 kg suspended by a light spiral spring extends it 8 cm when in equilibrium. A second mass of 2 kg is attached to the first without moving it and the combined mass is then released from rest.

- (i) Prove that the motion is simple harmonic.
- (ii) Find the periodic time of the ensuing motion.
- (iii) Find the maximum velocity of the resulting motion.

7. A particle of mass 10 kg is placed on a rough inclined plane. The least force acting up along the plane which will prevent the particle slipping down the plane is 19.6 N. The least force acting up along the plane which will make the particle slip upwards is 98 N.

- (i) Find the inclination of the plane.
- (ii) Show that the coefficient of friction is  $\frac{1}{2}$ .
- (iii) Find the least force required to move the particle up the plane.  
The least force need not necessarily be parallel to the plane.

8. Prove that the moment of inertia of a uniform circular lamina of mass  $M$  and radius  $r$  about an axis through its centre, perpendicular to the plane of the lamina, is  $\frac{1}{2} Mr^2$ .

A circular sheet of cardboard of radius  $r$  rotates freely in its own plane, which is vertical, about a horizontal pin. At what distance from the centre should the pin be stuck to make the period of small oscillation a minimum ?

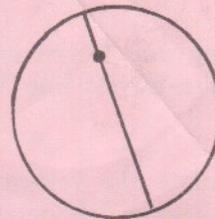


fig. 1

9. (a) A cube of side 1 m is filled to a height of  $x$  m with water and a second liquid of relative density 0.8, which does not mix with water, occupies the remainder of the cube. The thrust on each vertical side due to the water is equal to the thrust due to the other liquid. Find  $x$ .

- (b) Two solid uniform spheres each of radius 6 cm are connected by a light string and are completely immersed in a tank of water. The heavier sphere lies on the bottom of the tank. The relative densities of the spheres are 0.75 and 2.25 respectively. Find the tension in the string and the reaction between the bottom of the tank and the heavier sphere.

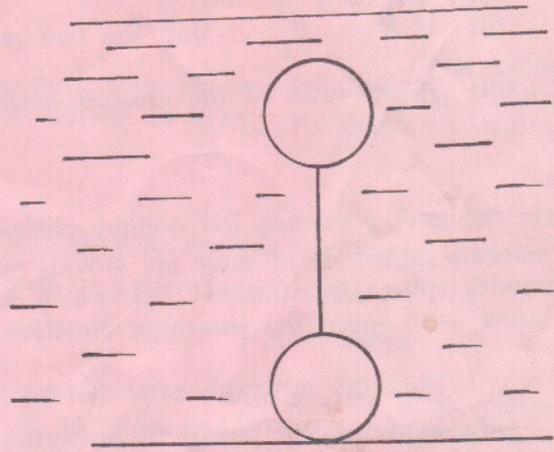


fig. 2

10. (a) Find the solution of the differential equation

$$x \frac{dy}{dx} = y + xy$$

if  $y = 1$  when  $x = 1$ .

$$x \frac{dy}{dx} = y(1+x)$$

$$\left( \frac{1}{y} dy = \frac{1+x}{x} dx \right)$$

$$\log_e y = \log_e x + x dx$$

- (b) A cyclist, free-wheeling on a straight level road, experiences a retardation which is proportional to the square of his speed. His speed is reduced from 6 m/s to 3 m/s in a distance of 35 m.

Show that the average speed during this period is  $6 \ln 2$ .