

APPLIED MATHEMATICS - HIGHER LEVEL

FRIDAY, 28 JUNE - AFTERNOON, 2.00 to 4.30

Six questions to be answered. All questions carry equal marks.
 Mathematics Tables may be obtained from the Superintendent.
 Take the value of g to be 9.8 metres/second². \vec{i} and \vec{j} are perpendicular unit vectors.

1. A bus 12.5 m long travels with constant acceleration. The front of the bus passes a point, p , with speed u while the rear of the bus passes p with speed v .

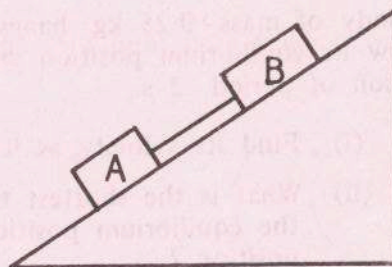
Find, in terms of u and v

- the time taken by the bus to pass p .
- what fraction of the length of the bus passes the point p in half this time.

2. A particle is projected from a point o with initial velocity u , up a plane inclined at an angle of 60° to the horizontal. The direction of projection makes an angle θ with the inclined plane. (The plane of projection is vertical and contains the line of greatest slope). The maximum height reached above the inclined plane is H .

- Express
- the velocity and displacement from o of the particle after t seconds, in terms of \vec{i} and \vec{j} , where these are the unit vectors along and perpendicular to the plane, respectively.
 - H , in terms of u and θ .
 - the time interval, in terms of $\sin 2\theta$, between the instants when the particle is at a height, $H \sin^2 \theta$, above the inclined plane.

3. Two blocks A and B have masses 2 kg and x kg, respectively. They are connected by a string and slide down an inclined plane which makes an angle $\sin^{-1}(\frac{3}{5})$ with the horizontal. The coefficient of friction between A and the plane is $\frac{1}{4}$, and between B and the plane is $\frac{1}{2}$.



- Show in a diagram the forces acting on each block when the system is released from rest.
- Find the acceleration f of the system in terms of x .
- For what value of x would the acceleration of the blocks be $0.9f$?

4. State the laws governing the oblique collision of elastic spheres.

A smooth sphere A impinges obliquely on an identical smooth sphere B which is at rest. The direction of A before and after impact makes angles 60° and θ , respectively, with the line of centres.

- (i) Prove that $\tan \theta = \frac{2\sqrt{3}}{1-e}$ where e is the coefficient of restitution between the spheres.
- (ii) Show that the maximum percentage loss in kinetic energy due to the impact is $12\frac{1}{2}\%$.
- (iii) For what value of e will the kinetic energies of A and B after impact be in the ratio $7 : 1$?

5. (a) Two cars A and B are moving along straight roads which are at right angles to each other, with uniform velocities 3 m/s and 4 m/s , respectively. When B is at the crossroads, A is 100 m away. Calculate the time interval for which the distance between the cars is not greater than 82 m .

- (b) A car of mass 750 kg attains a maximum speed of 30 m/s when travelling down an incline of 1 in 25 with the engine switched off. It can attain a maximum speed of 20 m/s up the same incline when the engine is working. The resistance to motion in each case is proportional to the square of the speed. Find

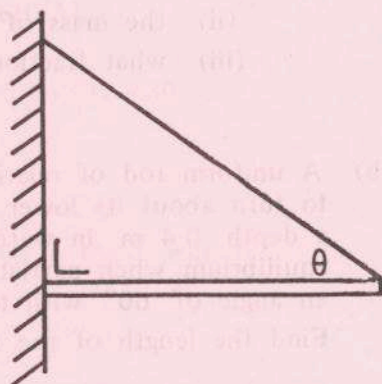
- (i) the power at which the engine is working
- (ii) the maximum speed of the car along a level road, if it works at the same power and its resistance is again proportional to the square of the speed.

6. Define simple harmonic motion.

A body of mass 0.25 kg hangs from a spiral spring. When pulled down 10 cm below its equilibrium position and released, it vibrates with simple harmonic motion of period 2 s .

- (i) Find its velocity as it passes through the equilibrium position.
- (ii) What is the shortest time taken to travel from a point 2 cm below the equilibrium position to a point 2 cm above the equilibrium position?
- (iii) Find the elastic constant of the spring.
- (iv) By how much will the spring shorten when the body is removed?

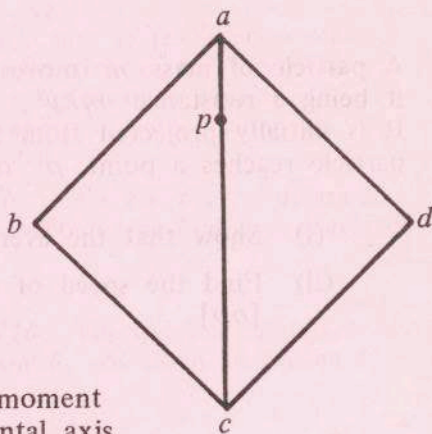
7. One end of a uniform metre stick of mass m is placed against a vertical wall. The other end is held by a light inelastic string making an angle θ with the metre stick. The coefficient of friction, μ , between the end of the metre stick and the wall is 0.4.



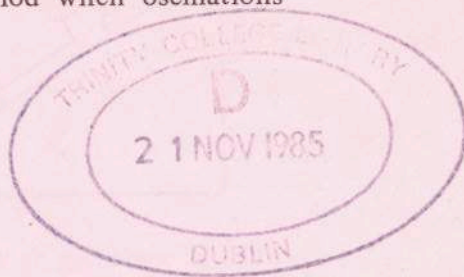
- (i) Show in a diagram the forces acting on the metre stick.
- (ii) Show that if the metre stick is to remain in equilibrium the maximum value of θ is given by $\tan \theta = \mu$.
- (iii) If a mass m is suspended from the metre stick at a distance x from the wall, show that the stick is on the point of slipping when

$$\tan \theta = \frac{2(1 + 2x)}{5(3 - 2x)}$$

8. A uniform square lamina $abcd$, of mass $3m$ and side $\sqrt{2}$, is free to rotate with its plane vertical about a smooth horizontal axis through a point p on the line ac . A mass m is attached at each of the points a and c .



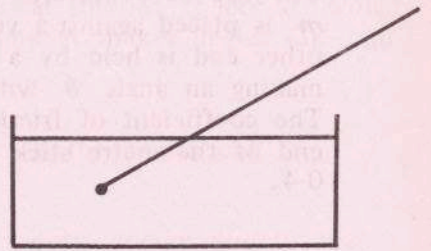
- (i) If $|ap| = 1 - x$, prove that the moment of inertia of the system about a horizontal axis through p is $m(3 + 5x^2)$.
- (ii) If the system oscillates about p , find in terms of x , the period for small oscillations.
- (iii) Find the value of x which gives the minimum period when oscillations are small.



OVER →

9. (a) A piece of gold-aluminium alloy of mass 10 kg, weighs 72 N in water. If the relative densities of gold and aluminium are 19.6 and 2.45 respectively, find
- the relative density of the alloy
 - the mass of each metal in the alloy
 - what fraction of the total volume of the alloy is gold.

- (b) A uniform rod of relative density 0.25 is free to turn about its lower end, which is fixed at a depth 0.4 m in water. The rod is in equilibrium when partially immersed and making an angle of 60° with the vertical. Find the length of the rod.



10. (a) Find the solution of the differential equation

$$3y^2(x - 1) \frac{dy}{dx} = 1 + y^3$$

if $y = 0$ when $x = 2$.

- (b) A particle of mass m moves in a straight line. The only force acting on it being a resistance mkv^2 , where v is its speed and k is a constant. It is initially projected from the point o with speed u . When the particle reaches a point p on the line its speed is $u/3$.

- Show that the average speed between o and p is $\frac{1}{2}u \log 3$.
- Find the speed of the particle when it is at the midpoint of $[op]$.