

**AN ROINN OIDEACHAIS
LEAVING CERTIFICATE EXAMINATION, 1974**

APPLIED MATHEMATICS — HIGHER LEVEL¹

THURSDAY, 28th JUNE — Morning 9:30 to 12

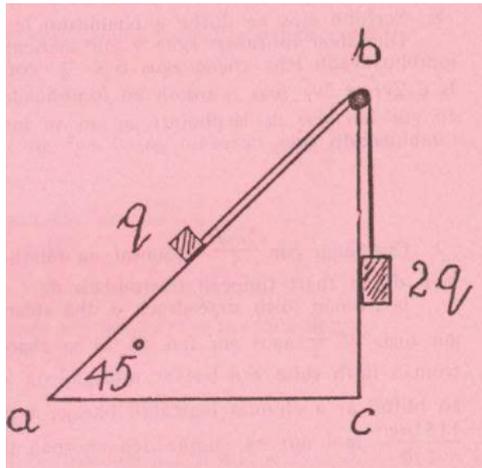
Six questions to be answered. All questions carry equal marks.

Mathematics tables may be obtained from the Superintendent.

Take the value of g to be 9.8 meters/second². \vec{i} and \vec{j} are perpendicular unit vectors.

1. A sprinter runs a race with constant acceleration k throughout. During the race he passes four posts a, b, c, d in a straight line such that $|ab| = |bc| = |cd| = 36\text{m}$. If the sprinter takes 3 seconds to run from a to b and 2 seconds to run from b to c , find how long, to the nearest tenth of a second, it takes him to run from c to d .
2. Two straight roads cross at right angles at p . On one road a car is travelling due north at 16m/s. As the car passes through p a bus is 100m from p and is travelling east towards p on the other road at 12m/s. Find the velocity of the bus relative to the car and show the relative path on a diagram containing the initial positions of the car and bus.
Calculate the least distance between the car and the bus in the subsequent motion and find when this occurs.
3. abc is the cross-section of a smooth heavy wedge of mass $4q$ where $|ac| = |cb|$ and $\angle acb$ is a right angle. The wedge rests with its face ac on a smooth horizontal table. A light inelastic string passes over a small smooth pulley fixed at the vertex b of the section of the wedge and supports particles of masses q , lying on the inclined plane ab and $2q$, hanging freely against the vertical face bc . When the system is released from rest, the wedge moves with an acceleration f parallel to ac and the mass q moves with an acceleration p relative to the wedge.

¹NB: Based on a transcription from Noel Cunningham's site and the Irish version of the paper.



Show on separate diagrams the forces acting on each particle and on the wedge, in particular the action of the string on the wedge. Write down the five equations of motion of the particles and the wedge involving f and p .

4. Define Simple Harmonic Motion in a straight line and show that if the magnitude of the displacement from the equilibrium position after time t is given by $x = a \sin(\omega t + \alpha)$, where a, ω, α are constants, then the motion is simple harmonic.

A light flexible elastic string pq , of natural length 1m and elastic constant 245N/m , has one end p tied to a fixed point and has a particle of mass 5kg attached to the end q . The particle is held 1m below p and is then released from rest to fall under gravity. By considering the forces acting on the particle when it has fallen a distance $(0.2 + x)\text{m}$, show that it moves with simple harmonic motion and that its acceleration is zero when $x = 0$.

Find the time taken to fall a distance 0.3m .

5. Show that $\vec{r} = c \cos \theta + c \sin \theta$, where c is a constant, is the position vector of a particle moving in a circle of radius c . Prove that the velocity \vec{v} is of magnitude $c\dot{\theta}$ at right angles to \vec{r} where $\dot{\theta} = \frac{d\theta}{dt}$. Find the components of the acceleration \vec{a} .

A particle of mass 0.2kg at c , is attached by two light inelastic strings ca and cb , each of length 1m , to a fixed point a and to a ring of mass 0.4kg at b , which is free to slide on a smooth fixed vertical wire ab . The particle and strings rotate about ab with constant angular speed $\omega = 10\text{rad/s}$.

Show on separate diagrams the forces acting on the particle and on the ring and calculate the tensions in the two strings.

6. Two uniform heavy rods pq and qr , each of length $2l$ and weight W , are freely joined together at q and hang freely from a fixed pivot at p . A force F acting in a horizontal direction is applied at r to the rod qr and an equilibrium position is reached when qr makes an angle of $\tan^{-1}(2)$ with the vertical.

In separate diagrams show the forces acting on pq and on qr .

- Find the horizontal and vertical components of the action in the hinge q and show that $F = W$.
- Prove that the magnitude of the reaction at p is W .
- If pq makes an angle α with the vertical, prove that $\tan \alpha = \frac{2}{3}$.

7. From a point p on a plane, inclined at $\tan^{-1}(\frac{1}{2})$ to the horizontal, a particle is projected with speed u at 45° to the plane. The motion takes place in a vertical plane through a line of greatest slope up the plane from p .

Express the velocity \vec{v} and displacement \vec{r} from p of the particle after time t in terms of \vec{i} and \vec{j} , where \vec{i} and \vec{j} are unit vectors along and perpendicular to the plane, respectively.

Prove that the particle strikes the plane at 90° and that the range on the inclined plane is $\frac{u^2\sqrt{5}}{4g}$.

8. State the laws governing the oblique collisions between smooth elastic spheres.

Two smooth spheres p and q of masses $2k$ and k , respectively, collide obliquely and the coefficient of restitution for the collision is $\frac{1}{2}$. The velocity of p before impact is $2v\vec{i} + 5v\vec{j}$ and the velocity of q before impact is $-4v\vec{i} + 3v\vec{j}$, where \vec{i} points along the line of centres at impact. Find the velocities of the spheres after the impact and show that the loss in kinetic energy is $9kv^2$.

9. Prove that the moment of inertia of a uniform solid sphere of radius a and of mass m about a diameter is $\frac{2ma^2}{5}$.

A symmetrical dumb-bell consists of two spheres joined by a narrow uniform rigid bar of mass $\frac{m}{2}$ and of length $2a$ so that the centres of the spheres are at a distance $4a$ apart. If the dumb-bell is freely pivoted at a point of the bar distance $\frac{a}{2}$ from its centre so that it can perform small oscillations in a vertical plane, prove that the moment of inertia of the body about the axis of rotation is $\frac{1151ma^2}{120}$ and find the period of oscillation.

10. A particle of mass 0.1kg falls vertically from rest under gravity in a medium which exerts a resisting force of magnitude $0.02v$ Newtons when the speed of the particle is $v\text{m/s}$. Show that

$$v = 49 \left(1 - e^{-\frac{t}{5}} \right)$$

and find $\lim_{t \rightarrow \infty} v$.

Find an expression for the distance travelled in time t seconds.