AN ROINN OIDEACHAIS

LEAVING CERTIFICATE EXAMINATION, 1972

APPLIED MATHEMATICS - HIGHER LEVEL

(400 marks)

FRIDAY, 23rd JUNE - MORNING, 9.30 to 12

Not more than six questions may be answered.
All questions are of equal value.
Mathematics Tables may be obtained from the Superintendent.
Take the value of g to be 9.8 metres/second^2.

1. A racing car covers a journey of 8·8 km from rest to rest. It accelerates uniformly in the first minute to reach its maximum speed of 40 m/s, it holds this speed for a certain time and then slows uniformly to rest with a retardation of magnitude three times that of the acceleration. Draw a rough velocity-time graph and find the distances travelled in the three stages of the journey and the total time taken.

If the maximum speed over the final kilometer of the journey had been restricted to 20 m/s, show that the time taken from rest to rest would have been at least 22·5 s longer than before, assuming the same rates of acceleration and deceleration as before.

2. From a point P on horizontal ground an elastic particle is projected under gravity with a velocity of $(28\vec{i} + 21\vec{j})$ m/s, where $\vec{i}$ and $\vec{j}$ are unit rectors along the horizontal and upward drawn vertical, respectively. Find the displacement $\vec{r}$ from p at any time t seconds afterwards, and in particular when the particle is at its highest point.

If at this point the particle strikes a fixed vertical wall, where the coefficient of restitution is $\frac{1}{2}$, find how far from p the particle strikes the ground.

3. State the laws governing oblique, perfectly elastic collision between two spheres.

A small sphere collides obliquely with a similar sphere of equal mass at rest - both spheres being smooth and perfectly elastic. Show that the paths of the two spheres after the collision are at right angles. Prove that there is no loss in kinetic energy.

4. A wedge of mass 2M rests on a smooth horizontal table with one of its plane faces inclined at 45° to the horizontal. This plane face is smooth and on it is placed a particle of mass M and the system is released from rest. Draw separate diagrams showing the forces acting on the particle and on the wedge during the motion. By considering the acceleration of the particle in two components, one component down the plane and the other horizontal, show that the acceleration of the wedge is $g/5$ and that of the particle is $g\sqrt{13}/5$.

5. Using the usual notation, prove that

$$v^2 = \omega^2(a^2 - x^2)$$

represents simple harmonic motion.

A light flexible elastic string of natural length 2 m and elastic constant 12 N/m has one end p tied to a point on a smooth horizontal table. To its other end is attached a particle of mass 3 kg, which is placed on the table at a point q, distance 3·5 m from p and released from rest. Show that in the first stage of the motion the particle moves with simple harmonic motion of period π seconds and that the time taken to reach p is

$(\frac{\pi}{4} + \frac{2}{3})$ s.

6. Show that a particle moving in a circle with constant speed is being accelerated towards the centre of the circle.

A small ring of mass 1 kg is threaded on a smooth light flexible inelastic string of length 0·8 m. The ends of the string are attached to two fixed points, distance 0·4 m apart in the same vertical line, and the ring describes with constant speed a horizontal circle whose centre is the lower fixed point. Find the constant speed of rotation and show that the tension in the string is 12·25 N.

7. A uniform rod of length 4 m and weight 100 N is smoothly hinged at one end to a rough horizontal floor. The rod rests on the smooth curved surface of a hemisphere whose plane face is on the floor. The rod is in equilibrium inclined at 45° to the horizontal and the hemisphere, of weight 50 N and radius 1 m, is in limiting equilibrium. Show in separate diagrams the forces acting on the rod and on the hemisphere. Find the reaction between the rod and the hemisphere, and prove that the coefficient of friction between the latter and the floor is $\frac{2}{3}$.

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8. Prove that the moment of inertia of a uniform rod *ab* of mass M and length $2l$ about an axis through *a*, perpendicular to the rod, is $\frac{4}{3}Ml^2$.

Such a rod is free to rotate in a vertical plane about a fixed horizontal axis at *a*, with a particle of mass 2M attached to the rod at *b*. The system is released from rest with the rod vertical and the end *b* above *a*. Show that the angular speed of the rod when it is next vertical is $(15g/7l)^{\frac{1}{2}}$.

At this point the particle falls off. Find the height to which the end *b* subsequently rises.

9. A particle of mass M kg is projected vertically upwards from ground level with a speed of 70 m/s. In addition to the weight of the particle, there is the resistance force of the air of magnitude $\frac{1}{20}Mv^2$ newtons when the speed is v metres per second. Show that the equation of motion during the upward journey is

$$-20\frac{dv}{dx} = \frac{v^2 + 196}{v}.$$

Prove that the maximum height reached is (10 ln 26) m, and that the time taken to reach it is approximately 1·95 s.

(Note: $\ln 26 = \log_e 26$)

10. State the conditions for the equilibrium of a floating body.

A uniform rectangular board *abcd* of weight W, floats with the diagonal *ac* on the surface of the water, the lowest corner *b* being attached to the bottom of the vessel by a light inelastic string. Show in a diagram the forces acting on the board and prove that the specific gravity of the board is $\frac{1}{3}$. Find the tension in the string.