AN ROINN OIDEACHAIS
LEAVING CERTIFICATE EXAMINATION, 1971

APPLIED MATHEMATICS — HIGHER LEVEL[1]

THURSDAY, 24th JUNE — Morning 9:30 to 12

Not more than six questions to be answered. All questions are of equal value.
Mathematics tables may be obtained from the Superintendent.

1. Explain how a graph of velocity plotted against time can be used to calculate acceleration and distance travelled, with particular reference to motion with constant acceleration.

   A pigeon in flight releases a small stone from its beak at a height of 50 metres when its velocity is u. If the stone takes $3\frac{1}{2}$ seconds to reach the ground, show that the direction of u is not horizontal and compute the greatest height reached by the stone after release. (Give your answer correct to the nearest tenth or 0.1 of a metre.)

2. A particle is projected from a point O on a plane inclined at 60° to the horizontal with a velocity $u = 7\sqrt{3}\vec{i} + 4.9\vec{j}$ metres/second where $\vec{i}$ is a unit vector through O pointing upward along the line of greatest slope in the plane and $\vec{j}$ is a unit vector perpendicular to the plane. Show that after time t seconds the position vector, $\vec{r}$ of the particle relative to O is given by

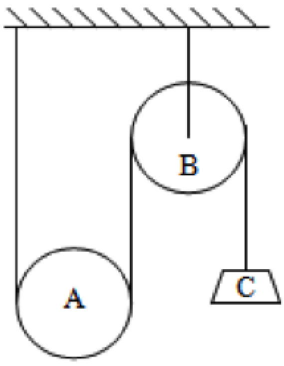
$$\vec{r} = \frac{7}{20}\left[(20\sqrt{3}t - 7\sqrt{3}t^2)\vec{i} + (14t - 7t^2)\vec{j}\right] \text{ metres.}$$

   Prove that the range on the inclined plane is $\frac{21\sqrt{3}}{5}$ metres, and find the velocity of the particle when it strikes the plane.

3. Two smooth spheres A and B of equal radii but of masses 20kg and 10kg respectively collide on a smooth horizontal table. Before collision the velocity of A is $(5\vec{i} + 3\vec{j})$m/s and the velocity of B is $2\vec{i}$m/s where $\vec{i}$ and $\vec{j}$ are unit perpendicular vectors in the plane of the table and $\vec{i}$ lies along the line of centres at impact. If the collision is perfectly elastic find the velocity of A and the velocity of B immediately afterwards.

4. A light inextensible string which is fastened at one end to a point in the ceiling passes under a smooth movable pulley A of mass 13kg, then over a smooth fixed pulley B, and a particle C of mass 9kg hangs freely from its other side. All parts of the string which are not touching the pulleys are vertical.

[1] NB: Based on a transcription from Noel Cunningham's site.



When the system is released from rest show that the acceleration of the particle is 2m/s^2 and that the acceleration of the pulley is half this. Find also the tension in the string.

5. Define simple harmonic motion in a straight line and show it can be described by the differential equation $\frac{d^2x}{dt^2} = -\omega^2 x$. Prove that $x = a\cos\omega t$, where a is a constant, is a solution of this equation.

   A particle describing simple harmonic motion on a straight line has velocities 4m/s and 2m/s when at a distance of 1m and 2m respectively from the centre of oscillation. Find the amplitude and periodic time of the motion.

   Calculate the least time taken for the particle to travel from the position of rest to the point where the velocity was 2m/s.

6. Show that $\vec{r} = a\cos\omega t\vec{i} + a\sin\omega t\vec{j}$, where a, ω are constants and $\vec{i}, \vec{j}$ are constant unit perpendicular vectors, is the position vector at time t of a particle moving in a circle. Show that velocity v is of magnitude aÏĽ and is at right angles to $\vec{r}$. Prove that the acceleration is of magnitude $a\omega^2$ and acts towards the centre of the circle.

   A particle of mass 10kg is describing a circle on a smooth horizontal table. It is connected by a light inelastic string of length 0.5m to a point which is 0.4m vertically above the centre of the circle. If the reaction of the table on the mass is 18N, calculate:

   (a) the constant angular velocity of the particle,

   (b) the tension of the string.

7. Two rods XY, YZ, of equal lengths but of weights $\frac{1}{2}W$ and W, respectively, are freely hinged together at Y. They stand in equilibrium in a vertical plane with the ends X and Z on a rough horizontal plane with the angle XYZ equal to $90°$. Show that the vertical components of the action of the plane at X and Z are $\frac{5W}{8}$ and $\frac{7W}{8}$ respectively.

   Show in separate diagrams the forces acting on each rod, using vertical and horizontal components for the reaction at Y. Find all these forces and prove that if one rod is on the point of slipping, the coefficient of friction is $\frac{3}{5}$.

8. Prove that the moment of inertia of a uniform circular lamina of mass M and radius r about an axis through its centre c, perpendicular to the plane of the lamina, is $\frac{1}{2}Mr^2$. Deduce that the moment of inertia about a parallel axis through a point of the circumference is $\frac{3}{2}Mr^2$.

   Such a lamina is free to rotate in a vertical plane about a horizontal axis perpendicular to its plane through a point a on its circumference. It is released from rest with ac horizontal. Find the angular velocity of the lamina when ac

makes an angle θ with the downward vertical, and show that the speed of c at its lower point is $\left(\frac{4gr}{3}\right)^{\frac{1}{2}}$.

9. State Archimedes Principle for a body immersed in a liquid. A uniform rod ab in equilibrium is inclined to the vertical with one quarter of its length immersed under water and its upper end a supported by a vertical force F.

   Show in a diagram the three forces acting on the rod, and prove that the specific gravity of the rod is 7/16. Express F in terms of W, the weight of the rod.

10. A particle of mass M is projected with speed u along a smooth horizontal table. The air resistance to motion when the speed of the particle is v, is Mkv where k is a constant. By solving the equation of motion for the particle, show that $v = ue^{-kt}$ and prove that as time increases indefinitely, the distance travelled ultimately approaches u/k. Find the time taken for the particle to travel half this distance.