Introduction to Sequences

Frequently in mathematics we are interested in patterns that appear in lists of numbers. Often we try to find a formula or a rule that would generate the list or pattern in the numbers. A sequence is a function whose domain is the natural numbers and we usually think of a sequence as a list of numbers. For example:

$$\begin{array}{cccccccc} 1, & 2, & 3, & . \ . \ . & n, & . & . & . \\ \downarrow & \downarrow & \downarrow & . \ . \ . & \downarrow & . & . & . \\ a_1, & a_2, & a_3, & . \ . \ . & a_n, & . & . & . \end{array}$$

The number in the n^{th} position on the list, i.e. a_n, is called the n^{th} term and the whole sequence is written as $\{a_n\}$.

A sequence can be finite or infinite and random or governed by some rule or formula. Consider the sequences below:

$$\begin{aligned} \{a_n\} &= \{2, 7, 27, 50\} \\ \{b_n\} &= \{2, 4, 6, 8\} \\ \{c_n\} &= \{3, 6, 9, 12, ...\} \end{aligned}$$

It is useful to be able to refer to a particular element in a sequence. So using the sequence $\{a_n\}$ for illustration, we refer to the first element in the list, 2, as a_1. The second element, 7, as a_2 and so on like that. (Similarly for the sequence $\{b_n\}$, we would refer to the first element, 2, as b_1. The second element, 4, as b_2 and so on like that also.) The numbers $\{a_1, a_2, a_3, ...\}$ are the terms of the sequence. For the above sequences we remark:

- The sequence $\{a_n\}$ is finite and there seems to be no obvious pattern.
- The sequence $\{b_n\}$ is finite and there does appear to be a pattern to the sequence. Each term is the previous term with 2 added to it.
- The sequence $\{c_n\}$ is infinite and there does appear to be a pattern to the numbers. Each term is the previous term with 3 added to it.

We said above that we are frequently interested in patterns that appear in sequences. It is often useful to try to find a formula to generate the sequence. Consider the sequence $\{c_n\}$ above. We said that the elements of the sequence are increasing by 3 each time. Hopefully you can see that we could generate all the terms in the sequence using the formula:

$$c_n = 3n$$

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Try the following exercises for practice.

(a) Let $\{a_n\}$ be a sequence where $a_n = n + 3$. Write out the first 10 terms of the sequence.

(b) Let $\{a_n\}$ be a sequence where $a_n = 2n - 3$. Write out the first 10 terms of the sequence.

(c) The first 5 terms of a sequence are given by $\{a_n\} = \{1, 4, 9, 16, 25, ...\}$ Find a formula for the n^{th} term of this sequence.

(d) The first 5 terms of a sequence are given by $\{b_n\} = \{4, 7, 10, 13, 16, ...\}$ Find a formula for the n^{th} term of this sequence.

(e) The first 5 terms of a sequence are given by $\{c_n\} = \{-1, 2, -3, 4, -5, ...\}$ Find a formula for the n^{th} term of this sequence.

Solutions

(c) $a_n = n^2$ (d) $b_n = 3n + 1$ (e) $c_n = (-1)^n n$