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# NOTES FOR TEACHERS

# MATHEMATICS

# GENERAL.

To teach these branches successfully it is necessary to have some clear conception of the end at which we are aiming. A certain amount of Arithmetical knowledge is necessary for everybody in the ordinary affairs of life, but a little reflection will show that it is small, and that the conveying of this amount of knowledge is not the main purpose of teaching even Arithmetic in the schools. We proceed beyond the mere essentials because of the training which the subject affords our pupils, training " in the art of thinking—consecutively, closely, logically," and also because of its application to the social life of the pupils.

We have to secure that the pupils are quick and accurate at simple calculations, which implies that they must have a thorough knowledge of the Tables and abundant practice at using them in the simpler operations of Arithmetic; that they study and understand principles so as to be able to apply them to the problems of life and, at the same time, to secure for themselves the maximum benefit in the way of clear thinking and preciseness and readiness of thought and expression. It is, then, the investigation or examination of principles, not the working of " Rules," that has to be undertaken; a sufficient number of simple concrete examples can be so presented that the underlying principle is readily grasped and then set out in a general statement. It is not suggested that every item must be grasped by every pupil before advancing: the progress of the class whose teacher insisted on this would be slow.

It is fortunate for the teacher that the aims set out above cannot be separated from one another: speed and accuracy in calculation as well as the investigation of principles can be developed through almost any kind of teaching material. The Department's Programme advises that in all mathematical work " *regard be had to the actual experience and interests of the pupils*," and it is well to consider what effect this should have on our selection of teaching material. Life is daily growing more complex and the teacher's task is likewise increasing daily in complexity; he has, therefore, constantly to keep adjusting the nature of his teaching material if he is to advance in step with the changes which are taking place in the larger world outside the schoolroom. It behoves him, then, to consider whether he cannot lighten his burden by discarding such things as Complex Fractions, and the endless exercises in so-called " Reduction," as well as Weights and Measures that have become obsolete, but which still figure in some school texts. Consider Troy weight, for instance: it may be of interest to give the senior pupils an exercise or two on the topic, the necessary *data* having been supplied, but it seems foolish in view of the demands made on the school from other directions, to occupy the time of a class for two or three weeks in working through the whole gamut of some particular group of weights or measures—Reduction Ascending and Descending, Addition, Subtraction, Multiplication and Division—returning to the topic again in Unitary Method and Practice, and memorising a table that will afterwards be found to be useless. It may be mentioned that recently a class of grown girls in a city school was found calculating the price of *nine gold cups*, each weighing etc., etc.; they were asked to give an estimate of their own weights, and only one ventured a reply—she thought she weighed half a ton. Again, in a rural school beside a creamery, a class of senior pupils were found operating on *circulating* decimals; but they did not know whether the creamery bought its milk supplies by the pound or by the gallon, and it surprised themselves and their teacher that the visitor should want to know how many gallons of milk would make 1 lb. of butter and how many lbs. a gallon of milk weighs. These—and many similar cases might be adduced—are instances of how far the school work may be removed from reality. Another aspect of the tendency towards unreality is exemplified by the following exercises (they have *not* been specially prepared for the purpose of these Notes):—

(1) Reduce 9,876,543 sq. ins. to acres.

| | ac. | rd. | p. | yd. | ft. | ins. |
|---|---|---|---|---|---|---|
| (2) From | 2 | 2 | 0 | 0 | 2 | 113 |
| take | | 3 | 0 | 39 | 8 | 139 |

(3) A man works 7 mos. 3 wks. 5 dys. 14 hrs. at 1s. 3½d. an hour. How much does he earn?

These have an artificiality all their own. The first and second have clearly no practical value, and if the division by 30¼ is the reason for the existence of (1) it could surely have been more effectively dealt with by a different collocation of numbers, and, therefore, of ideas. The third example is worthy of closer consideration: consider the mental state of the pupil who has been to such an extent *doped*—there is no other word for it—by repeated aimless work in "Reduction," as to handle this question on the assumption that 1 month=4 weeks, 1 week=7 days, 1 day=24 hrs. Yet it was so worked by a class of pupils and marked off as correct by the teacher—in modern times. Considerable profit might have been derived by showing, if the pupils themselves were not ready to suggest it, that the *data* are inadequate, and then having the problem re-stated.

If this aspect of the subject appears to be unduly stressed here—and a great deal more might be said on the matter—it is not only on account of the serious waste of time and energy involved but still more on account of the deadening effect which such an unintelligent way of employing the young pupil's time and energy must have on his developing mind.

It should be noted, with regard to the "actual experience and interests" of the pupils, that even in the Infants' class no pupil comes to school without some knowledge of number and being able to count to some extent. This knowledge should be utilised from the beginning. Except where the aim is speed and accuracy—which demand abundant practice at abstract calculation and depend to a large extent, as has already been said, on an accurate knowledge of the tables—the pupils will be engaged in the working of "real" problems (and it should be remembered that problems that are "real" to young people may not appear so to adults). They will weigh and measure, and the simpler weights and measures and boxes of imitation coins will be in the school; the dimensions of the schoolroom and playground, ascertained by the pupils themselves, will form material for further real problems; they will have some idea of the acreage of a neighbouring field, and of what a mile really means (pupils have been known in Standard VI. to estimate as a mile what was really about 100 yards); only weights and measures actually in use will be taught—and these whether they are in the Table Book or not, e.g., the "weight" of potatoes, the "peck" of coals, the number of "fingers" to a yard, etc.

Nor would it be unreasonable that problems worked in schools in rural areas should involve such *data* (or *desiderata*) as the quantity of potatoes or oats necessary to sow a Statute or Irish acre, and the resulting produce in tons or barrels, and its value at current price per ton, or per barrel.

If the 25 in. Ordnance sheet is in the school, with acreages marked to three decimal places, it can give all the necessary practice in manipulating decimals—addition, subtraction, multiplication, division, and changing over to roods and perches (with the further exercise, if the pupils are ambitious, of changing the Statute measure over to Irish measure).

The Creamery Book, the Savings' Certificate Associations, the Railway Guide are mines from which adequate material may be procured for all operations involving averages, percentages, rates of travel, etc., etc. In the Seventh and Eighth Standards the question of local "rates" could, and should, be touched on.

Here, for instance, is an extract from a "Statement of Monthly Account" from a creamery:—

| | |
|---|---|
| (1) Quantity of Milk supplied .. .. .. | 8536 lbs. |
| (2) Percentage of Butter Fat therein .. .. | 3.65 |
| (3) Equivalent price per gallon .. .. .. | 7.72d. |
| (4) Total lbs. of Butter Fat supplied .. .. | 311.5 |

The "Statement" contains further information not of use for our purpose. Taking items (1) and (4), we ask, "If 8536 lbs. of milk produce 311.5 lbs. of butter, how many lbs. of butter will be got from 100 lbs. of milk?" (Ans. 3.65.) And again, "If 100 lbs. of milk produce 3.65 lbs. butter, how much butter from 8536 lbs.?" (Ans. 311.5 lbs.)

Next: "If this butter were sold at 1s. 8½d. per lb. how much did it realise?" (Ans. is in the body of the "Statement," viz., £26 12s. 3d.) The creamery manager uses a Ready Reckoner: the pupils also should be taught how to use one.

Further: "What is meant by 'Equivalent price per gallon'?" "If the total value was £26 12s. 3d. at 7.72d. per gallon of milk, how many gallons were there?" "Now calculate how many pounds of milk make a gallon."

Nor are we finished yet. "Separated milk is returned to the extent of 80 per cent. of the new milk supplied; if it is worth ¾d. a gallon to the farmer, what is the total return to him for the month, reckoning butter fat and separated milk supplied?"

Surely this kind of exercise has greater interest for the pupils, and greater practical value, and a greater value as training—for all the purposes in fact for which we teach Arithmetic—than selling cows at £16 11s. 7¾d. each, and papering rooms with paper 21 inches wide costing 2s. 6d. *a yard*.

Here is an extract from a local newspaper:—

"The average milk yield of these 274 cows was 7,605 lbs. milk and 274.54 lbs. butter fat. Taking butter fat at 1s. per lb., and separated milk at 1d. per gallon, the average return per cow of 590 gallons would be £16 3s. 8d. The average for the ten best cows in the Association is 12,141 lbs. milk and 443.51 lbs. butter fat. Taking butter fat at 1s. per lb. and separated milk at 1d. per gallon, the total return per cow would be £26 1s. 11d. The best cow in the Association, a 9-year old, yielded 14,199 lbs. milk, and 504.06 lbs. butter fat; two cows average 14,120 lbs.; four cows average 12,327 lbs., eight cows average 11,487 lbs., and ten cows 10,615 lbs."

And this from a daily paper:—

"A BIG PROFIT.—He said beet seed would be given each farmer at 10d. the lb., which would amount to £1 per Irish acre, and it would be deducted when the farmer delivered the beet to the factory. With an average crop of 16 tons to the acre, average sugar content 17.5, the factory would pay £47 4s. per acre for the roots. The farmers would have the crown and leaves, representing on an average 12 tons or about £9, making a total of £56 4s.; cost of growing, about £25; clear profit £30 the acre. The price was guaranteed for three years."

"If the yield were 15.5 the farmer would receive an extra 2s. 6d. per ton, and for 17.5 the factory would pay 59s."

Paragraphs like these may be handled in the manner already indicated. The matter in them is not arranged in as orderly and clear a manner as could be desired, and needs to be carefully examined with a view to separating the independent data and supplying those that happen to be missing. Examination of the figures in the first extract reveals, for instance, that the 590 gallons are gallons of separated milk; and that additional data are required—the weight in pounds of a gallon of new milk and the percentage (80) of the new milk returned as separated milk. If the figures are anywhere discrepant, so much the better perhaps—it gives an opportunity for the exercise of ingenuity in detecting the discrepancy and gives a warning which the ready-made question of our text-books (with an answer that usually must work out even) never gives—that statements given in printed figures as well as in printed words (especially in newspapers) need to be carefully weighed.

The text book cannot supply all this: even the best text must take a comprehensive view and cannot be expected always to supply the local bias that is needed. A good teacher will, therefore, use his text-book with discrimination, and will not think himself bound either to follow its order in the presentation of matter or to work every exercise it contains: he will omit unsuitable matter or questions, or modify the questions by using smaller numbers or by altering the conditions of a problem. A skilful teacher could, of course, especially if he had only one standard to teach, dispense with it altogether; but to do so is hardly feasible in the vast majority of cases—for practice in the mechanical side of the work it seems to be essential. In Arithmetic a text should be selected for the class which contains a properly graded selection of exercises only—without answers and without either explanatory matter or worked examples. The worked examples are often useless and occasionally harmful, while the explanatory matter serves no purpose (except, perhaps, for a private student)—the pupils do not read it, and would not understand it if they did.

There is no point in the practice which sometimes prevails of separating Arithmetic into Oral and Written. If it is remembered that ordinarily we have recourse to paper only when (1) the numbers are too large to carry in our minds or (2) the process too complicated, it will be understood that oral work should have a large share of attention, not only for the sort of work which does *not* fall under these two categories, but for finding an approximation to a result before working the problem in detail. Thus if the pupil were asked—before working the question on paper—to say *about* how much the worker in Q. 3 on page 2 above would earn in the given period, the query would prevent the egregiously absurd type of answer which frequently makes its appearance in examinees' papers, and should, in this instance, have the effect of making the pupil ask how many hours per day the man is expected to work. At all events, oral work will come first; the new principle or process will be made clear in the first instance by abundant exercises with small numbers; these will be made larger till the need for using pen and paper forces itself on the scholar. For these preliminary oral exercises a textbook is not necessary (it is rather a handicap), and if this method is consistently adopted, there will be no great need for setting apart a special period for "Mental Arithmetic." And the method employed in the two cases will be generally the same. Thus 300 lbs. of butter at 1s. 2d. a lb. gives (orally) 300s. (or £15)+50s., that is, £17 10s., which is the ordinary "practice" method; and, if the interest on £300 for 2 years at 4 per cent. gives £3×2×4 by oral method, so the interest on £365 10s. for 2 years at 4 per cent. will appear on paper as £3.655×2×4, setting down the Principal in hundreds, after having decimalised it.

In the senior classes written work should be done in ink and neatly. The practice of drawing every line with a ruler is a serious source of time-wasting, and distracts the pupil from the matter in hand; lines should be drawn freehand. Ruling off a column for what is known as "Rough Work" provides an excuse for untidy and careless work; there should be no *rough* work—whatever calculations incidental to the question are necessary (and the calculations made on paper are often more than is necessary—even advanced students are still found dividing by 100 on paper and by long division !) should be set out neatly and clearly at the side where they will not interfere with the general "thread" of the solution. The pupil's arrangement of his matter should, in fact, be as easily read (by one conversant with the symbols) as an ordinary piece of prose. If this is done, the teacher's revision of the pupils' written work—a troublesome business, but necessary—is considerably simplified.

Revision of previous work is, of course, necessary; but if "topics" (such as the Creamery question) are occasionally dealt with, revision becomes automatic. In some cases, e.g., Fractions in a particular standard, the work would naturally begin with a review of the work done in the preceding standard.

## THE WORKING OF THE PROGRAMME.

The early language lessons will have an important bearing on the number work. All teachers will have noticed that pupils who do not come to school till 6 or 6½ years, though they are behind the others in formal school work, Reading and Number, soon make up the deficiency, through their wider vocabulary and greater interest in things in general. Hence it would appear that the language work, even when it is the mother-tongue, should be to some extent directed towards giving the pupils the vocabulary and the ideas necessary for the early work in number—greater than, less than ; long, short ; high, low ; add, take away ; half, quarter ; square, circle ; names of coins, etc., etc. In any case, the language lessons cannot proceed very far until counting becomes necessary and the children count in games, skipping, for instance. Also there will be rhymes with counting. Formal work in number should develop from these—counting first in ones, then in twos, threes, fives, tens, not only to form a basis for the multiplication table, but to clear the way for an appreciation of place values ; if, ordinarily, we count in tens, we count days in sevens, pence in twelves, and so on.

In regard to the earlier stages a word of warning appears necessary against

(1) dealing too soon with *abstract* numbers ; the process of abstraction is a difficult one, and should be gradual, with frequent reversion to the concrete ;

(2) putting the pupils to *written* Arithmetic too soon; the sequence of thought gets broken and the effort to write well tires the pupil ;

(3) the tendency to proceed too fast in the beginning.

The tables should be built up by the pupils themselves—the pupils still manipulating the objects—and afterwards memorised, and teachers of Standards I. and II. should remember that the success of the work in the upper standards depends in a large measure on the manner in which the tables are dealt with in the lower standards, especially the Multiplication table.

It is suggested that written addition be introduced as an equation

$$3+4=7,$$

and that subtraction be taught under each of these forms :—

$$8-3=5 \text{ (direct subtraction)},$$
$$3+?=8 \text{ (inverse addition)}.$$

Multiplication will be taught as continued addition,

$$2+2+2=2\times3,$$

and $2\times3$ should be always read as $2+2+2$, i.e., as *three times* two, not twice three ; and the Multiplication table had better be built up on this form :—

$$3=3\times1=\ 3 \text{ (once three)},$$
$$3+3=3\times2=\ 6 \text{ (twice three)},$$
$$3+3+3=3\times3=\ 9 \text{ (three times three)},$$
$$3+3+3+3=3\times4=12 \text{ (four times three)},$$

rather than three ones, three twos, etc., which is

$$1+1+1=1\times3=\ 3$$
$$2+2+2=2\times3=\ 6$$
$$3+3+3=3\times3=\ 9$$
$$4+4+4=4\times3=12.$$

Division, like Subtraction, will be taught under two aspects :—

(1) divide 28 into 4 equal parts—sharing ;

(2) how often can 7 be taken out of 28 ?—which is repeated subtraction.

In formal written addition pupils should be trained to repeat the totals only ; thus for units, 11, 15, 18 and the figure to be carried should be written at the top of the column to which it belongs, that is, if it is found necessary to write it at all. In multiplication teachers should consider the advisability of beginning with the left hand digit of the multiplier ; it is the most important figure and one avoids having to reverse the procedure later, when contracted methods are taught. In Division arrangements such as the following should be avoided :—

$$\begin{array}{r} 13 \\ 4 \\ 5 \\ 16 \\ \hline \end{array}$$

$$3)\underline{482}$$
$$160+2$$

it suggests that the quotient is, perhaps, 162 which is the normal interpretation of 160+2. Better write " 160 and remainder 2," and later $160\frac{2}{3}$ when the pupils' knowledge of fractions is adequate.

Written problems should not be introduced until the pupils can read with ease. It will be appreciated by all teachers that the grasping of the facts of such problems presents serious difficulty to young students. Even advanced students are sometimes found presenting at examinations a solution to a problem differing materially from the one actually set, and the primary school pupil is frequently found working out a solution to a problem of his own making which he has read into the question in his text-book. The first step, then, is training in grasping the conditions or facts of the problem, and it is a very valuable training ; then the consideration of how best to use these facts in order to arrive at the information for which the problem asks ; next the actual working out of the result, with whatever check the nature ot the question enables one to apply.

Teachers will be well advised to keep the range of their work in the lower standards within the limits as to notation set out in the Programme. The tyranny of the text-book is probably responsible for the tendency to use large numbers ; but they merely weary the pupils and nothing is gained. If one considers that problems in addition and subtraction are, normally, merely repetitions of the process of adding a number *less than ten* to a number *less than a hundred*, it will be seen that the manipulation of the large numbers brings no advantage.

In dealing with fractions in Standard II. it is not desirable that the pupils should use, or even see, the symbols ½, ⅓, ¼ ; if they write these fractions, or if the teacher writes them on the blackboard, it is the words that should be written. Even in Standards III. and IV. it is 3 fifths, 5 sevenths, etc., which should be written—at least in the early stages. Further, the teacher should have it in his mind at this stage that he, or somebody else, will have at a later stage to teach ratio, and that the work now should be so directed as to leave nothing to be undone at the later stage.

Compound Addition, Subtraction, Multiplication, Division should be treated as the simple rules were. It should be explained that we must now count the pence in twelves (not in tens) and the shillings in twenties (not in tens). There is no new principle to apply ; there is merely a variation in the process. Nor is there any reason why, for instance, feet and inches should not be introduced at the same time as shillings and pence, and tons and cwts., along with pounds and shillings—in 2nd or 3rd Standard.

It has been found that, in Standard IV. the bulk of the time and attention has been given to subhead (*d*) of the Programme to the comparative neglect of (*b*), (*c*), (*e*), (*f*). It should be observed in this connection that "Reduction" is not mentioned in the Programme neither for Standard IV. nor elsewhere. Exercises on what is called, for want of a better name, "reduction," will, of course, be worked ; but they must be treated rationally. The whole range of tons, cwts., qrs., lbs., ozs., does not occur together in everyday affairs, and there should be no place in the school work for finding how many tons, etc., in 2,000,000 ozs. It is suggested that, if the pupils were exercised in arranging their work after the following fashion,

18 tons 5 cwts. 4 stones=365 cwts. 4 stones,
=2,924 stones ;

and the reverse process thus

2,924 stones =365 cwts., 4 stones,
=18 tons, 5 cwts., 4 stones,

instead of always adhering to the ladder arrangement, their ideas as to the significance of the processes would be clearer.

The warning against using unduly large numbers in the earlier standards needs to be repeated in Standards V., VI., i.e., against carrying decimal calculation to undue lengths. The three places of decimals mentioned for Standard V. will be found adequate for all ordinary purposes. Consider that .001 of £1 is less than ¼d., and it is clear that when the pupil sets down, say, £5 14s. 8.3615 as the answer to a question he is using figures that are meaningless. He should be trained to write instead £5 14s. 8.4d., or £5 14s. 8½d.

The treatment of fractions seems to present the greatest difficulty to teachers of senior standards, and no pains should be spared in giving the pupils clear ideas as to their meaning and manipulation. For concrete representation of fractions, geometrical diagrams appear to be the most satisfactory. Thus a circle divided into four equal parts by a pair of perpendicular diameters shows at once $\frac{2}{4}=\frac{1}{2}$; another pair of diameters interposed symmetrically between the first pair gives $\frac{4}{8}=\frac{2}{4}=\frac{1}{2}$; while three diameters drawn at equal angular intervals give $\frac{3}{6}=\frac{1}{2}$, $\frac{2}{6}=\frac{1}{3}$, etc., or a square or rectangle might be similarly used. Carefully drawn freehand diagrams are probably best as the process is essentially one of reasoning rather than of measurement.

As soon as the principle of "equivalent" fractions is made clear addition and subtraction of fractions present no difficulty, and one may proceed to deal with multiplication and division on such lines as these :—

Since $\frac{2}{7}+\frac{2}{7}=\frac{4}{7}$ and $\frac{2}{7}+\frac{2}{7}+\frac{2}{7}=\frac{6}{7}$, it follows that $\frac{2}{7}\times 2=\frac{4}{7}$ and $\frac{2}{7}\times 3=\frac{6}{7}$ and that $\frac{1}{2}$ of $\frac{4}{7}=\frac{2}{7}$ and $\frac{1}{3}$ of $\frac{6}{7}=\frac{2}{7}$.

Again, since $\frac{1}{3}$ of $\frac{6}{7}=\frac{2}{7}$, it follows that $\frac{2}{3}$ of $\frac{6}{7}=\frac{4}{7}$. Now, if we wish to ascertain how much is $\frac{1}{3}$ of $\frac{5}{7}$ we have $\frac{1}{3}$ of $\frac{5}{7}=\frac{1}{3}$ of $\frac{5\times3}{7\times3}=\frac{1}{3}$ of $\frac{15}{21}=\frac{5}{21}$; and, to ascertain how often $\frac{1}{3}$ is contained in $\frac{5}{7}$, we write

$$\frac{\frac{5}{7}}{\frac{1}{3}}=\frac{\frac{15}{21}}{\frac{7}{21}}$$

from which it appears that we are required to ascertain how often seven units (of a new kind) are contained in fifteen units, and it follows that

$$\frac{\frac{5}{7}}{\frac{1}{3}} \text{ is equivalent to } \frac{15}{7}.$$

So far the symbolism employed has avoided the use of the ordinary sign of multiplication where the multiplication of two fractions is concerned. The word "of" should continue to be used for a considerable time and a number of examples worked from first principles before any attempt is made to embody the results in a rule. After the pupils have worked, for instance, a number of examples of the type

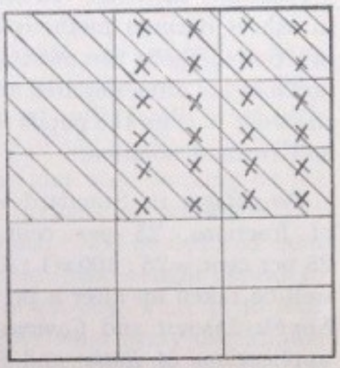
$$\frac{1}{3} \text{ of } \frac{5}{7}=\frac{1}{3} \text{ of } \frac{5\times3}{7\times3}=\frac{1}{3} \text{ of } \frac{15}{21}=\frac{5}{21}$$

they will begin to see that the two intermediate steps may be omitted and that the final result may be obtained by merely multiplying the two numerators and also the two denominators.

Before the ordinary sign of multiplication is used some consideration should be given to its meaning ; hitherto the symbol has implied continued addition, thus

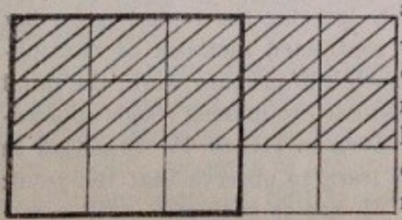
$$\frac{2}{3}\times 4=\frac{2}{3}+\frac{2}{3}+\frac{2}{3}+\frac{2}{3}$$

i.e., $\frac{2}{3}$ added four times, an idea which is not applicable to the symbolism $\frac{2}{3}\times\frac{3}{5}$.



Continuing with the idea of $\frac{2}{3}$ of $\frac{3}{5}$, we note that the lined portion of the rectangular area in the diagram is $\frac{3}{5}$ of the total area ; and that the crossed area is $\frac{2}{3}$ of the lined ; counting rectangles we find that $\frac{2}{3}$ of $\frac{3}{5}$ of the whole area contains six of the smaller units, each of which is $\frac{1}{15}$ of the whole. Hence $\frac{2}{3}$ of $\frac{3}{5}=\frac{6}{15}$. Now, adverting to the manner in which the area of a rectangle has been found when dealing with whole numbers, it will easily be seen that, if the rectangular area above is a square of unit side, the doubly-shaded portion is a rectangle whose dimensions are, respectively, $\frac{2}{3}$ and $\frac{3}{5}$ of the linear unit ; and, if the principle already established, for finding the area of a rectangle when the dimensions were integers holds for fractional numbers also, the area of the doubly-shaded rectangle ought to be $\frac{2}{3}\times\frac{3}{5}$. But counting rectangles shows that this area is 6 fifteenths. Hence he may write

$$\frac{2}{3}\times\frac{3}{5}=\frac{6}{15}.$$



Division may be illustrated as follows. Begin by inquiring with regard to a given unit of area—the rectangle in the diagram —what fraction $\frac{3}{5}$ of the area is of $\frac{2}{3}$ of it ? The shaded portion represents $\frac{2}{3}$ of the unit, and the heavy outline encloses $\frac{3}{5}$ of it, and counting of squares reveals that the required result is 9 tenths.

This brings us back to the process already employed for reducing the fractions to equivalent fractions having a common denominator ; the diagram shows at once that $\frac{2}{3}$ is equivalent to $\frac{10}{15}$, and $\frac{3}{5}$ to $\frac{9}{15}$, that we are, in other words, inquiring how often $\frac{9}{15}$ is contained in $\frac{10}{15}$. It is quite likely that it would be an advantage to avoid the terms G.C.M. and L.C.M., certainly in the early stages. There is a good deal of evidence that many pupils who use these terms, even in seventh standard, have only a hazy notion of what is meant by a Multiple and none at all of what is meant by a *common* Multiple. In all the processes the treatment should be very simple; a great deal of the time devoted to the manipulation of complex fractions could be employed more profitably.

The remainder of the Arithmetical work consists chiefly of applications of fractions, vulgar and decimal, with an intelligent use of "Practice" methods. In some cases fractional methods are easiest; in others decimal methods are least cumbersome; while in others practice methods are most suitable. Judicious combinations of the methods are often required and an effort should be made throughout the work to train the pupils to use the method which gives the neatest and readiest solution.

*Percentages* in Standard V. should be connected up with equality of fractions, 25 per cent.$=\frac{25}{100}=.25=\frac{1}{4}$, and, later, with Ratio, 25 per cent.$=25:100=1:4$, and if Geometry is taught *Ratio* could well be taken up after a preliminary investigation of similar figures. *Simple Interest* and *Commercial Discount* will be treated as direct applications of *Ratio*, and *Profit and Loss* questions should not be (as is usual in text-books) segregated into a separate compartment. In problems on Profit and Loss pupils are often confused as to the quantity of which they ought to reckon the percentage. A discussion of the difference between saying that A is 5 per cent. less than B, and B is 5 per cent. more than A, so as to bring out the fact that usage requires that in the first we are concerned with 5 per cent. of B, and in the second with 5 per cent. of A, would lessen the confusion. It is very desirable that the meaning of *Compound Interest* be illustrated by working a few simple examples, if only to give the pupils an understanding of "growth problems," such as increase of population. *Stocks and Shares* are very unreal for young people, and the primary school is justified in entirely ignoring the topic. A further word as to *approximations*: while contracted methods of multiplication and division will not enter much into primary work, the pupil should, from an early stage, be accustomed to making rough approximations as a check on his work. Even in III. or IV. Standard in working, say, £3.3.8$\times$52, he should learn to observe that the result must exceed £150. If the pupil of VI. has to evaluate

$$\frac{589\times3.142}{83.76}$$

he should be trained to observe that this quantity is roughly equal to

$$\frac{600\times3}{80}$$

i.e., approximately 20.

This not only saves him from blundering answers but enables him to ignore decimal points altogether in the calculation, as he knows now where to place the decimal point in the result.

## GEOMETRY.

It appears desirable at the outset to say something as to the position of Euclid.

Euclid wrote for adults—some 2,000 years ago—what has been described as "a collation and systematic arrangement of the Geometric knowledge of his time," and this in itself should be enough to make us doubtful as to the suitability of his methods for introducing the subject to young pupils. His aim was, working on *the smallest possible number of assumptions* (axioms or postulates) to *prove everything, no matter how obvious*, which could be deduced from simpler assumptions ; this is a process which makes slight appeal to the young pupil, who sees little point in proving what is to him intuitively evident—"You ask him to prove what every dog knows, that the sum of two sides of a triangle is greater than the third side." Again, Euclid's system provided a set proof for every proposition, the pupil being expected to memorise this proof, and no suggestion was offered as to how Euclid, or anybody else, might have arrived at the solution ; it is clear that one does not learn effectively how to reason well by memorising samples of the reasoning of another, no matter how sound the latter's work may be, and, though the "riders" or exercises added to Euclid's work by successive editors helped to remedy this weakness in his system, it is to be feared that the majority of the students were so occupied in memorising Euclid's proofs of his propositions that the amount of time and attention given by them to original investigation or independent reasoning was negligible.

It may be pointed out that about thirty years ago it was permitted by some examining bodies to break away to some extent from the Euclidean tradition. Teachers of Geometry were allowed to depart from Euclid's sequence of propositions, and "hypothetical constructions" were tolerated, that is one might assume, for instance, that a given finite straight line had a middle point before proving the construction for determining this point. These concessions, already an admission that Euclid's methods were not quite the best for beginners, simplified somewhat the earlier stages of the work in Geometry, and it is now universally recognised that Euclid's system, as an introduction to the study of Geometry, is gone for ever.

We differ, then, from Euclid in the larger body of assumptions which we use. No attempt will be made to prove the obvious, or what appears to the pupil to be intuitively evident. If the pupil's intuition leads him to accept what is false, it is easy to demonstrate to him by a concrete example or by a special case that his assumption or statement is false. Again, we differ from Euclid in this, that not only will the proof or demonstration not always be supplied, but even the conclusion to be reached may be suppressed. Thus, instead of asking the class to "show that the figure formed by the four bisectors of the interior angles of a parallellogram is a rectangle," we prefer to say, "If you bisect the four internal (or external) angles of a parallelogram, what is the nature of the figure formed by the bisectors?" It will be suggested that the figure in question is a parallelogram or a rectangle, or, possibly, a square; reasons will be required for these statements until it is made clear that one of them is correct as far as it goes, but incomplete, that another is wrong, and that the remaining one is true for every parallelogram. Later one proceeds by inquiring what the result would be if the original figure were itself a rectangle? Or a square? And if, at this stage, a pupil inquires "What about the case where the original figure is neither a parallelogram, nor a rectangle, nor a square—just a quadrilateral?" the teacher may assume that things are going well. But, instead of answering the question, he should set the class to work on it if their attainments in Geometry warrant such a course; if not, the pupil will be advised to bring the question up later. Thus help will be given only when needed and in the smallest possible degree. The pupils will learn to demonstrate by demonstrating on their own account. They will be "engaged in exploring and investigating the facts" of Geometry for themselves, and the teacher will prevent them from going astray by asking reasons for their statements or inferences, and these reasons will be examined for their relevancy or adequacy. The arrangement, classification and inter-relation of the results of the pupils' investigations as well as the building up of suitable definitions will come at a later stage when they have become familiar with a certain body of facts and processes and are sufficiently mature to appreciate the characteristics of a good definition.

Discretion has to be exercised with regard to the extent to which numerical measurements may be used as demonstration. When the pupil is required to construct a triangle and to determine the sum of its angles by direct measurement with a protractor, the only inference that can be drawn from the process is that, for that particular triangle, the sum of the three angles is—to a greater or less degree of approximation—180 degrees; if a number of triangles be so treated the probability of the correctness of the inference increases with the number of cases tried, but there is still only probability. For this particular point as well as for the equality of vertically-opposite angles and the angle properties of parallel lines, the pupils can, by a simple demonstration with rotating rods, be led to accept them as facts.

In written examinations, the actual " form " in which the demonstration is cast is of minor importance : what the examiner looks for is evidence, clearly and concisely set out, of the candidate's grasp of facts and principles.

A beginning should be made with the notions already in the pupil's possession. Before reaching Standard V. he will have some notion of a line, of a rectangle (usually called an oblong in his paper-folding exercises), and of a right angle (which he has, perhaps, been taught to call a square corner). He will have drawn plans in his Geography work and discussed the points of the compass, and will, therefore, have some notion (however vague) of an angle. All this will be reviewed ; from the cardinal points proceed to the clock-hands. Reference will be made to the manner in which the gardener lays out a circular flower-bed ; how he secures that his cabbages are sown in a straight line ; how the carpenter secures that his corner is square ; how the mason secures that his wall is vertical ; and " horizontal " and " vertical " will be discussed. There will be some practical work in constructing ornamental designs with ruler and compass. Loci will be introduced as arising out of concrete local problems. Thus a field beside a straight road has a well whose distance (in the shortest line) from the road is 120 yards, where may I build a house so that, say, a particular point in it may be 40 yards from the road and 100 yards from the well ? Where may I build it so that it may be *at least* 40 yards from the road and at least 100 yards from the well ? So that it may be *not more* than 40 yards from the road and *not more* than 100 yards from the well ? etc., etc.

Special pains should be taken to secure that pupils have some clear notion of what an angle is and of a right angle. Pupils asked to draw on the blackboard an angle greater than a given angle have been known to lengthen the legs of the angle—and they were older than 11–12 years ; and numerous are the pupils who, when asked what they mean by a right angle, promptly reply " 90 degrees," and when pressed as to what is a *degree* answer 1-90th of a right angle. The idea is not sufficiently stressed that the right angle is the *unit*. Also when the right angle is explained as the angle between two lines, one horizontal and one vertical drawn through the same point, one is surely justified in inquiring how the terms " horizontal " and " vertical " are explained.

The work of Sixth Standard will begin by a study of symmetry, congruence of triangles, angle-properties of parallel lines, similar figures. It is not necessary that they should be taken in that order, nor that all four should be dealt with before taking up some of the work indicated under par. (*g*) of the Syllabus. It is, in fact, very necessary, that every teacher should prepare for himself a scheme of work setting out in detail the matter with which he proposes to deal during the year; such scheme to be amended or modified from year to year according as experience suggests.

As to Symmetry, most pupils will have at one time or another folded a sheet of paper, and, having made a sketch in ink at one side of the crease, have folded the sheet back on itself while the ink was still wet—to astonish himself and his neighbours with the "symmetry" of the result. A start might be made on this. Study the manner in which the "image" was formed so as to make it clear that:

(1) the line joining any two "corresponding" points is perpendicular to the line of the crease, i.e., to the axis of symmetry;
(2) the shortest distance, i.e., the perpendicular distance, of any point from the axis is equal to the perpendicular distance of its image from the axis;
(3) the distance between any two points is equal to the distance between their two "images."

Having examined for symmetry each of the figures suggested at paragraph (*b*) refer back to the practical constructions in paragraph (*a*) of the programme for Standard V., and draw attention to the axes of symmetry. The pupil will appreciate the fact that, in the ordinary construction for the bisection of an angle, e.g., when each leg of the angle is treated in exactly the same way, the resulting figure cannot but be symmetrical.

Congruence of triangles is usually approached by constructing triangles from given data:

(1) Given two sides and the included angle;
(2) ,, three sides;
(3) ,, one side and two angles;
(4) ,, three angles;
(5) ,, two sides and an angle other than the included angle.

Examine whether the solution is in each case "unique" (the term need not be used), i.e., whether in each case more than one triangle can be constructed. Cut them out and superimpose them if you will, but their coincidence is not a ***proof*** of their congruency. It will be found, for instance, under (1) that all the triangles constructed having sides 2 in. and 3 in. and the included angle $70^\circ$ are in no way different one from another, that they are in fact, the same triangle; if so, we shall henceforth assume that any two triangles in which the parts $a_1, b_1, C_1$, are identically the same as $a_2, b_2, C_2$, have the remaining sides and angles identically equal. The fourth case will be put aside for further consideration under similar figures.

Reference will again be made back to the constructions carried out under (*a*) of Standard V. and the pairs of congruent triangles discovered; the reason for their congruency in each case—whether (1), (2) or (3) above; and the inference drawn that, from this point of view also, the constructions have achieved their purpose.

In dealing with parallelism one begins also with the ideas already possessed by the pupil. Even if he has not heard the word at all, he sees parallels everywhere, in the flooring-boards, in the uprights of the door, in the edges of his desk, etc. Having directed his attention to these instances and inquired for others in order to make sure that he grasps what the teacher is looking for, the latter proceeds to elicit the pupil's idea of parallelism. If the pupil identifies parallelism with "the same distance apart," one makes that the starting point and proceeds to establish the angle properties. If the pupil identifies parallelism with the "direction" idea, one proceeds to examine what is meant by "direction," and a start is made with the assumption that two parallel lines make equal angles with some third line. The treatment of the subject may, in fact, be based on any consistent idea that is in the pupil's mind: he should not be forced at the outset to accept some new concept which belongs to somebody else's pre-arranged plan.

From the first day when the child attempted to make a drawing he was showing his acquaintance with similar figures; in his work on maps and plans he went a considerable step further. There will be little difficulty now in getting him to grasp the fact that if there are, e.g., two maps of Ireland, on different scales, the following relation is true:—

$$\frac{\text{Distance from Dublin to Cork}}{\text{Distance from Dublin to Galway}} = \frac{\text{Distance from Dublin to Cork}}{\text{Distance from Dublin to Galway}}$$

or that corresponding linear dimensions are, for the two maps, in a constant ratio. The triangles constructed under (4) at page 16 can now be re-introduced; they are of the same shape and the linear dimension-ratio again holds. To demonstrate that the areas of such triangles are proportional to the squares of their linear dimensions will not be a difficult matter after the area of a triangle has been shown to be

$$\frac{\textit{base} \times \textit{altitude}}{2}.$$

$$\frac{\Delta_1}{\Delta_2} = \frac{\frac{1}{2}a_1\ p_1}{\frac{1}{2}a_2\ p_2}$$

$$= \frac{a_1}{a_2} \times \frac{p_1}{p_2}$$

$$= \frac{a_1}{a_2} \times \frac{a_1}{a_2} \left(\text{since } \frac{p_1}{p_2} = \frac{a_1}{a_2}\right)$$

$$= \frac{a_1^2}{a_2^2}.$$

And generally that

$$\frac{\triangle_1}{\triangle_2}=\frac{a_1^2}{a_2^2}=\frac{b_1^2}{b_2^2}=\frac{c_1^2}{c_2^2}=\frac{d_1^2}{d_2^2}$$

where $d_1$, $d_2$ are any two corresponding dimensions, e.g., two medians, two bisectors of corresponding angles, etc.

Now, using all or any of these results, i.e., Symmetry, Congruence, Angle-properties of parallel lines, Similar Figure properties (and it will be noted that these are not " proved " in the sense of being " deduced " from something more fundamental than themselves) we proceed to consider the matter contained in (*g*). The extent to which this matter will be studied will depend on the character of the school. It will usually include

(*a*) the angle-property of the triangle, and its extension to figures of more than three sides; the in-centre and circum-centre, treated as intersection of loci; concurrency of medians;

(*b*) area of the triangle, leading, later to the area of the circle; equality of areas of triangles on same (or equal) bases and with equal (or same) altitude; area of parallelogram (equal to area of " corresponding " rectangle); equality of areas of parallelograms on equal (or same) bases and having equal altitudes (between the same parallels);

(*c*) theorem of Pythagoras;

(*d*) angle properties of the circle; the cyclic quadrilateral; the orthocentre of a triangle; reference should be made to circles on the Globe and to Latitude;

(*e*) drawing of tangents to circles;

(*f*) the equality of certain groups of areas treated algebraically, e.g.,

$$x(a+b+c)=ax+bx+cx,$$
$$(a+b)^2=a^2+2ab+b^2,$$
$$a^2-b^2=(a+b)(a-b),$$
$$(a-b)^2=a^2+b^2-2ab.$$

In the more favourably circumstanced schools the rectangle property of intersecting chords of a circle might be examined in connection with similar figures; the case where the point is outside the circle would be considered, including the special case where one of the chords reaches the limiting position of a tangent. Here, also, the work might be extended to cover a few simple problems in heights and distances as, for instance, ascertaining the height of a tree or steeple by comparing lengths of shadows or ascertaining (by drawing a plan to scale) the distance between two remote objects, when the distance of each from a given point is known, as well as the angle between the lines along which these distances are measured, or the distance between two inaccessible objects—by any method (again using a plan drawn to scale). Also such problems as these which really belong to the domain of plane geometry :—

Finding the length of the diagonal of a rectangular block, a brick, for instance ;

Finding the area of a section of a rectangular block which passes through two diagonally opposite edges ;

Finding the plane surface of a right pyramid whose base is a square of given side and whose altitude is known (all of which involve a direct application of the theorem of Pythagoras) ; and this :

To find the area of a right section of a circular cone where the section divides the altitude in a given ratio.

It should be noted that propositions dealing with inequalities are difficult, and, if selected for demonstration, they should be approached carefully.

## ALGEBRA.

A generation ago " Elementary Algebra " was merely a collection of rules for manipulating masses (sometimes very large) of symbols. That neither the rules nor the symbols themselves were understood is indicated by the number of intelligent educated people who tell us that they had, when at school, " no head for Mathematics " ; if they only knew what the " $x$ " was, all would be well. Half a century ago, Fitch encouraged teachers to practise the habit of " embodying any truth you have ascertained (in Arithmetic) in the shape of a formula in which the letters of the alphabet are substituted for numbers." He gives an example :—

" Equal additions to two numbers do not alter their difference." He would illustrate this by simple examples like

$$12-7=5\,;$$

hence $$(12+8)-(7+8)=5\,;$$

and then proceed to generalise thus :

$$\text{if } a-b=c \text{ then } (a+n)-(b+n)=c.$$

" But," he adds, " do not suppose that this is Algebra—it is merely the statement of an Arithmetical truth in its most abstract form. It lifts the pupil out of the region of particulars into the region of universal truths. It helps him to see that what is true of certain numbers . . . is necessarily true of all numbers." And he goes on to recommend " the practice of embodying each Arithmetical truth, as you arrive at it, in a general formula." Whether this is " Algebra " or not, it provides a very useful transition from the Arithmetical work. We shall, then, accustom our pupils to generalise :—

(1) $\frac{1}{2}=\frac{3}{6}=\frac{7}{14}=\frac{n}{2n}$, where $n$ is any number ;

(2) or, having shown that

$$\frac{3}{11}+\frac{4}{11}+\frac{1}{11}=\frac{8}{11},$$

then

$$\frac{3}{x}+\frac{4}{x}+\frac{1}{x}=\frac{8}{x};$$

(3) having shown that the area of a rectangle four feet by five feet is

$$A=4\times 5$$

teacher will proceed to suggest that if the length were $l$ feet and breadth $b$ feet, then

$$A=l\times b.$$

(There will be present to the teacher's mind in these generalisations that "any number" means here any positive integer ; the pupils' minds will also be so interpreting the term.)

So they will have from the earliest stages a grasp of what the "$x$" means.

The first item, the study of the "meaning and use of Arithmetical symbols" in the Standard V. programme is usually neglected. It would, perhaps, be well to begin by glancing at the Roman numerals with some of which most pupils are familiar from the clockface ; the arrangement whereby

VI=V+I=5+1,
IV=V—I=5—1,
XXI=X+X+I=10+10+1,

will be considered, nor would it be without interest for the pupils to consider how the Roman pupil performed these operations :—

XXIII+XVII=?
XXXV—XIX =?
XXXV×XVI =?

and to infer that with such a system of Notation progress in Mathematics, or even in Arithmetic, was impossible. The ordinary arithmetical symbols will then be reconsidered as to their use and meaning.

$$\text{That } 35=10\times3+5=3 \text{ tens } + 5\,;$$
$$\text{while } 53=10\times5+3=5 \text{ tens } +3$$

will be examined to bring out the fact that what is primarily involved here is *addition*. This is very necessary in view of the fact that presently the pupils will be taught to write

$2x$ to signify $2\times x$
$xy$ ,, $x\times y$

where the juxtaposition of the symbols now implies multiplication.

The simple equations used in "solving easy problems" ( (*b*) of Programme) will be of the simplest kind: problems will be dealt with which give rise to equations like

$$5n+2=17.$$

It is not always observed that in connection with (*c*), exercises in addition, subtraction, multiplication, division, are confined to quantities consisting of a *single term*. Thus

$$3x+4x-x=?$$
$$8n^2-3n^2=?$$
$$\frac{8ax}{4a}=?$$
$$2a\times3a=?$$
$$2ax\times4ax=?$$

Teachers would even be well advised to confine the first exercises to those where only a single letter is involved.

There is no serious objection to the working of exercises involving binomial quantities or even expressions of three terms such as

(1) Add $2a+3x$
and $5a+2x$

(2) From $3a+4x$
take $2a+x$

or (3) Multiply $x+4$
by $x+5$

provided signed numbers are not introduced—operations with these latter should be postponed till (*c*) of Standrard VI. programme has been studied. (1) and (2) here are, in fact, simpler than the corresponding operations with pounds and shillings; while (3) is essentially the same operation as that involved in multiplying 24 by 25.

In Standard VI. "further practice in the meaning and use of symbols" will include the manipulation of formulae. Pupils will *construct* formulae :—

1. Area of rectangle : $A = l \times b$.

2. ,, triangle : $A = \frac{1}{2}a.p.$

3. Formula for S. Int. : $I = \frac{P \times R \times T}{100}$

4. Distance covered by train at uniform rate : $D = v.t.$

5. A formula giving the length (in yards) of carpet *n* in. wide that will cover a floor *a* ft. by *b* ft.

6. A formula for all numbers which leave a remainder 5 when divided by 7 etc., etc. They will write (2) in these forms also ·

$$p = \frac{2A}{a} \text{ and } a = \frac{2A}{p}$$

In its original form the formula was suited to finding the area of a triangle when the base and altitude were known ; it is now suited to finding, first the altitude when the area and base are known, and secondly the base when area and altitude are known. Such work will be of considerable help in translating the language of problems into the Shorthand of Algebra.

A warning is necessary against the manipulation of "nests" of brackets. Brackets are introduced by mathematicians only when they need them ; and they are removed when they happen to be in the way. This should suggest their proper use : if one wishes to indicate that $b - c$ is to be subtracted from $a$, one writes $a - (b - c)$, but if it becomes necessary to subtract $2ab$ from $3a(b+c)$, the bracket is now in the way and is removed. The "operations with algebraic expressions containing two or three terms" will include ordinary addition, subtraction, multiplication, and division, as well as addition, subtraction, multiplication, and division of fractions ; L.C.M. need not be even mentioned in dealing with addition and subtraction of fractions and in every case there will be reference back to the corresponding operation in Arithmetic. If H.C.F. and L.C.M, are dealt with, it will be by the method of factors.

The pupils' knowledge of factors will have been advancing from the first exercises in multiplication and division. A couple of simple exercises in Multiplication, e.g.,

(1) multiply $x+4$ by $x+5$ (2) multiply $x+3$ by $x+7$

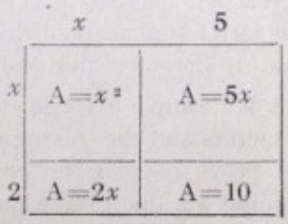
will enable the pupil to *write down* the product of

$$(x+2)(x+3)=?$$
$$(x+6)(x+8)=?$$
even $$(x+\tfrac{1}{2})(x+\tfrac{1}{4})=?$$

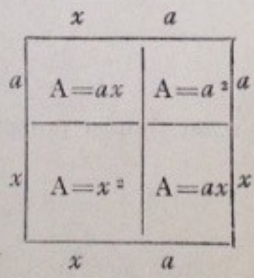
and to generalise $(x+a)(x+b)=x^2+(a+b)x+ab$.

Geometric illustration—even for pupils who do not study Geometry—could conveniently be resorted to here :—

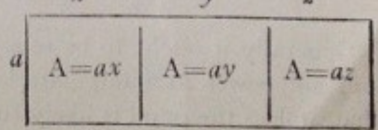
$$(x+5)(x+2)=x^2+5x+2x+10$$
$$=x^2+7x+10$$




$$(x+a)^2=x^2+2ax+a^2$$




$$a(x+y+z)=ax+ay+az$$




The last paragraph, "positive and negative numbers and operations with these," will call for full and careful treatment. The pupils' range of numbers has, up to this point, been confined first to whole numbers (positive integers) and then to fractions (still positive), and the minus sign has been used solely as the symbol of subtraction. Difficulties with subtraction force negative numbers on his notice and the minus sign will be given a new part to play. Thus, a man walks five miles east and then three miles west, how far is he from his starting point? (5 — 3, or 2 miles.) If he walks " $a$ " miles east and " $b$ " west? (Ans. $a - b$ miles.) If " $b$ " is greater than " $a$ " what does this mean? The ***meaning*** of *signed* numbers will be thoroughly investigated by discussion of problems such as the above dealing with direction, problems connected with assets and liabilities, with rise and fall of thermometer, or rise and fall of tide etc. Concrete work in addition, subtraction, multiplication and division will follow. The previous work will then have to be reviewed so as to include exercises involving such numbers. It is desirable that the difficulty of operating with these symbols should be separated from the difficulty involved in operating with the "generalised" numbers, which constituted the algebraic symbols dealt with up to this point.

Teachers having any doubt as to procedure in connection with this latter topic cannot do better than consult chaps. x., xi. Algébar, Cuid I. (Browne and Nolan).

A few simple exercises in square root might follow the work done in factors and the solving of a few simple quadratic equations should not prove beyond the powers of an ordinary Sixth Standard boy. Finding a value of $x$ which will make

$$x^2 - 7x + 12 = 0,$$

$$\text{or} \quad x^2 + 12 = 7x,$$

$$\text{or} \quad 7x - x^2 = 12,$$

is a far simpler proposition in some ways than solving

$$\frac{x}{3} + \frac{3x}{5} + \frac{5}{11} = \frac{1}{2}x,$$

which is called a "simple" equation.

Generally it ought to be remembered that it is grasp of principles, and not power to manipulate large masses of symbols, that counts; that, unless the pupil is likely to specialise in Mathematics, the purely manipulative work can easily be carried too far, e.g., divide $x^3 + y^3 - z^3 + 3xyz$ by $x + y - z$, and its cousin, multiply

$$x^2 + y^2 + z^2 - yz - zx - xy \text{ by } x + y + z$$

may well be left to those who are likely to specialise in Mathematics.

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