AN ROINN OIDEACHAIS

THE LEAVING CERTIFICATE

MATHEMATICS SYLLABUS

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THE LEAVING CERTIFICATE

MATHEMATICS

CONTENTS

1. INTRODUCTION

1.1 CONTEXT

Mathematics is a wide-ranging subject with many aspects. On the one hand, in its manifestations in terms of counting, measurement, pattern and geometry it permeates the natural and constructed world about us, and provides the basic language and techniques for handling many aspects of everyday and scientific life. On the other hand, it deals with abstractions, logical arguments, and fundamental ideas of truth and beauty, and so is an intellectual discipline and a source of aesthetic satisfaction. These features have caused it to be given names such as "the queen and the servant of the sciences". Its role in education reflects this dual nature: it is both practical and theoretical -- geared to applications and of intrinsic interest -- with the two elements firmly interlinked.

Mathematics has traditionally formed a substantial part of the education of young people in Ireland throughout their schooldays. Its value for further and higher education, for employment, and as a component of general education has been recognised by the community at large. Accordingly, it is of particular importance that the mathematical education offered to students should be appropriate to their abilities, needs and interests, and should fully and appositely reflect the broad nature of the subject and its potential for enhancing the students' development.

1.2 AIMS

It is intended that mathematics education would:

A. Contribute to the personal development of the students:
- helping them to acquire the mathematical knowledge, skills and understanding necessary for personal fulfilment;
- developing their modelling abilities, problem-solving skills, creative talents, and powers of communication;
- extending their ability to handle abstractions and generalisations, to recognise and present logical arguments, and to deal with different mathematical systems;
- fostering their appreciation of the creative and aesthetic aspects of mathematics, and their recognition and enjoyment of mathematics in the world around them;
- hence, enabling them to develop a positive attitude towards mathematics as an interesting and valuable subject of study;

B. Help to provide them with the mathematical knowledge, skills and understanding needed for life and work:
- promoting their confidence and competence in using the mathematical knowledge and skills required for everyday life, work and leisure;
- equipping them for the study of other subjects in school;
- preparing them for further education and vocational training;
- in particular, providing a basis for the further study of mathematics itself.

It should be noted that in catering for the needs of the students, the courses should also be producing suitably educated and skilled young people for the requirements of the country.

1.3 GENERAL OBJECTIVES

The teaching and learning of mathematics has been described as involving <u>facts</u>, <u>skills</u>, <u>concepts</u> (or "conceptual structures"), <u>strategies</u>, and -- stemming from these -- <u>appreciation</u>.

In terms of student outcomes, this can be formulated as follows. The students should be able to <u>recall</u> relevant facts. They should be able to demonstrate <u>instrumental understanding</u> ("knowing how") and necessary <u>psychomotor</u> skills. They should possess <u>relational understanding</u> ("knowing why"). They should be able to <u>apply</u> their knowledge in familiar and eventually in unfamiliar contexts; and they should develop <u>analytical</u> and <u>creative</u> powers in mathematics. Hence, they should develop <u>appreciative</u> attitudes to the subject and its uses. The aims listed in Section 1.2 can therefore be translated into general objectives as given below.

<u>Fundamental objectives</u>

A. Students should be able to <u>recall</u> basic facts; that is, they should be able to:
- display knowledge of conventions such as terminology and notation;
- recognise basic geometrical figures and graphical displays;
- state important derived facts resulting from their studies.

(Thus, they should have fundamental information readily available, to enhance understanding and aid application.)

B. They should be able to demonstrate <u>instrumental understanding</u>; hence they should know how (and when) to:
- carry out routine computational procedures and other such algorithms;
- perform measurements and constructions to an appropriate degree of accuracy;
- present information appropriately in tabular, graphical and pictorial form, and read information presented in these forms;
- use mathematical equipment such as calculators, rulers, setsquares, protractors, and compasses, as required for the above.

(Thus, they should be equipped with the basic competencies needed for mathematical activities.)

C. They should have acquired <u>relational understanding</u>, i.e. understanding of concepts and conceptual structures, so that they can:
- interpret mathematical statements;
- interpret information presented in tabular, graphical and pictorial form;
- recognise patterns, relationships and structures;
- follow mathematical reasoning.

(Thus, they should be able to see mathematics as an integrated, meaningful and logical discipline.)

D. They should be able to <u>apply</u> their knowledge of facts and skills; that is, they should be able when working in familiar types of context to:
- translate information presented verbally into mathematical form;
- select and use appropriate mathematical formulae or techniques in order to process the information;
- draw relevant conclusions.

(Thus, they should be able to use mathematics and recognise it as a powerful tool with wideranging areas of applicability.)

E. They should have developed the psychomotor and communicative skills necessary for the above.

F. They should appreciate mathematics as a result of being able to:
- use mathematical methods successfully;
- acknowledge the beauty of form, structure and pattern;
- recognise mathematics in their environment;
- apply mathematics successfully to common experience.

Other objectives

G. They should be able to analyse information, including information presented in unfamiliar contexts:
- formulate proofs;
- form suitable mathematical models;
- hence select appropriate strategies leading to the solution of problems.

H. They should be able to create mathematics for themselves:
- explore patterns;
- formulate conjectures;
- support, communicate and explain findings.

I. They should be aware of the history of mathematics and hence of its past, present and future role as part of our culture.

Note

Many attempts have been made to adapt the familiar Bloom taxonomy to suit mathematics education: in particular, to include a category corresponding to "carrying out routine procedures" ("doing sums" and so forth). The categories used above are intended, *inter alia*, to facilitate the design of suitably structured examination questions.

1.4 PRINCIPLES OF COURSE DESIGN

To implement all the aims and objectives appropriately, three courses were designed: one (the Higher course) at Higher level, and two (the Ordinary and Ordinary Alternative courses) at Ordinary level. The two Ordinary level courses are complementary to each other, catering for different populations with different interests, needs, and learning styles. The Ordinary Alternative course was introduced into schools in 1990, and is sanctioned for examination up to and including Summer 1994.

The following principles influenced the design of all courses.

A. They should provide continuation from and development of the courses offered in the Junior Cycle.

Hence, for the cohort of students proceeding from each Junior Cycle course, there should be clear avenues of progression. These should take account of the background, likely learning style, potential for devlopment, and future needs of the target group.

B. They should be implementable in the present circumstances and flexible as regards future development.

They should therefore be teachable, learnable and adaptable.

(a) They should be teachable, in that it should be possible to implement the courses with the resources available.
- The courses should be teachable in the time normally allocated to a subject in the Leaving Certificate programme.

Thus, they should not be unduly long.
- Requirements as regards equipment should not go beyond that normally found in, or easily acquired by, Irish schools.
- They should be teachable by the current teaching force. Hence, the aims and style of the courses should be ones that teachers support and can address with confidence, and the material should in general be familiar.

(b) They should be <u>learnable</u>, by virtue of being appropriate to the different cohorts of students for whom they are designed.
- Each course should start where the students in its target group are at the time, and should proceed to suitable levels of difficulty and abstraction.
- The approaches used should accommodate different abilities and learning styles.
- The material and methods should be of interest, so that students are motivated to learn.

(c) They should be <u>adaptable</u> -- designed so that they can serve different ends and also can evolve in future.
- A measure of choice can be provided, both within courses (by providing "options" while requiring coverage of basic and important material), and between courses (by recognising the need for different types of course).
- New material (in the "options") can be tried in the classroom and maybe later moved to the core, while material with lessening relevance can be phased out gradually.
- Appropriate responses can be made as resource provision changes (for example, allowing more emphasis on use of computers).

C. <u>They should be applicable, preparing students for further and higher education as well as for the world of work and for leisure</u>.
Where possible, the applications should be such that they can be made clear to the students (now, rather than in some undefined future), and hence ideally should be addressable at least to some extent within the course.

D. <u>The mathematics they contain should be sound, important and interesting</u>.
A broad range of appropriate aspects of mathematics should be included.

1.5 <u>NOTE</u>
The Computer Studies option, which has been listed with Mathematics in <u>Rialacha agus Clár</u>, is not addressed in this booklet; so provision described here does not affect the running of the option. It should be noted, however, that computers (equipped with appropriate software packages) may of course be used in the teaching and learning of Mathematics.

2. HIGHER LEVEL

2.1 INTRODUCTION

One course is offered at Higher level.

2.2 HIGHER COURSE: RATIONALE, STYLE AND AIMS

The Higher course is aimed at the more able students. Students may choose it because it caters for their needs and aspirations as regards careers or further study, or because they have a special interest in mathematics. The course should therefore equip mathematical "specialists" -- students who will pursue advanced mathematics courses; but it should also cater suitably for students who will not proceed to further study of mathematics or related subjects. Hence, material is chosen for its intrinsic interest and general applicability as well as its provision of concepts and techniques appropriate for future specialists in the field.

Students who follow the Higher course will already have shown their ability to study mathematics in an academic environment. The course offers opportunities for them to deepen their understanding of mathematical ideas, to encounter more of the powerful concepts and methods that have made mathematics important in our culture, and to enhance their enjoyment of the subject.

For the target group, particular emphasis can be given to aims concerned with problem-solving, abstracting, generalising and proving. Due attention should be given to maintenance of the more basic skills, especially in algebra (where students shortcomings have been seen to stand in the way of their own progress). However, it may be assumed that some aims regarding the use of mathematics in everyday life and work have been achieved in the Junior Cycle; they are therefore less prominent at this level.

2.3 HIGHER COURSE: PRIOR KNOWLEDGE

Knowledge of the content of the Junior Certificate Higher course will be assumed.

2.4 HIGHER COURSE: ASSESSMENT OBJECTIVES

The assessment objectives are the fundamental objectives A, B, C, D and E (see Section 1.3), interpreted in the context of the following aims of the Higher course:

- a deepened understanding of mathematical ideas;
- an appreciation of powerful concepts and methods;
- the ability to solve problems, abstract, and generalise, and to prove the results specified in the syllabus (marked with an asterisk(*));
- competency in algebraic manipulation.

Note:

As indicated by objective E, the students should present their work comprehensibly; this is especially relevant when they are using calculators.

2.5 HIGHER COURSE: STRUCTURE AND CONTENT

The syllabus is presented in the form of a core and a list of options. It is envisaged that students would study the whole of the core and one option.

CORE
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Note on examination of "bookwork"

In the case of results marked with an asterisk (*), formal proofs may be examined; in the case of other results stated in the syllabus, proofs will not be examined.

Algebra

1. Algebraic operations on polynomials and rational functions. Addition, subtraction, multiplication and division and the use of brackets and surds.
   Use of Remainder theorem not required.

   Laws of indices and logarithms.

   *The Factor Theorem for polynomials of degree two or three.

   Factorisation of such polynomials (the linear and quadratic factors having integer coefficients).

   Solution of cubic equations with at least one integer root.

   Sums and products of roots of quadratic equations.
   Cubics excluded

2. Unique solution of simultaneous linear equations with two or three unknowns.
   Sets of equations with non-unique solutions or no solutions excluded.

   Solution of one linear and one quadratic equation with two unknowns.

3. Inequalities: solution of inequalities of the form $g(x) < k$, $x \in R$, where $g(x) = ax^2 + bx + c$ or $g(x) = \frac{ax + b}{cx + d}$

   Use of the notation $|x|$; solution of $|x - a| < b$

4. Complex numbers: Argand diagram; addition, subtraction, multiplication, division; modulus; conjugate; conjugates of sums and products; conjugate root theorem.
   Properties of modulus excluded.

   *De Moivre's theorem: proof by induction for $n \in Z$; applications such as nth roots of unity, $n \in Q$, and identities such as $\cos 3\theta =$

$4 \cos^3\theta - 3 \cos \theta$.

Loci <u>excluded</u>. Transformations from z-plane to w-plane <u>excluded</u>.

5. *Proof by induction of simple identities such as the sum of the first n integers and the sum of a geometric series, simple inequalities such as $n! >= 2^n$, $2^n >= n^2$ $(n>=4)$, and $(1+x)^n >= 1+nx$ $(x>-1)$, and factorisation results such as: 3 is a factor of $4^n - 1$.

6. Matrices: dimension, 1x2, 2x1 and 2x2 matrices; addition; multiplication by a scalar; product.

   Properties: addition of matrices is commutative; multiplication of matrices is not necessarily commutative.

   Identities for addition and multiplication. Inverse of a 2x2 matrix.

   Application to solution of two linear equations in two unknowns.

Transformations <u>excluded</u> from this section. For further applications, see section on Geometry.

<u>Geometry</u>

Results from the Junior Certificate Higher course may be assumed; proofs will not be required.

1. Line:

   General equation of line in form $ax + by + c = 0$.

   *Length of perpendicular from (x_1, y_1) to $ax + by + c = 0$.

   *Angle θ between two lines with slopes m_1 and m_2 $\left(\tan \theta = \dfrac{m_1 - m_2}{1 + m_1 m_2}\right)$.

   Equation of line through the intersection of two lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ {form $L(a_1x + b_1y + c_1) + M(a_2x + b_2y + c_2) = 0$, L,M constant}.

   Division of a line segment in ratio m:n.

Parametric equations of line (confined to verification that given parametric equations represent a line).

Pairs of lines *excluded*.

Circle:

Equation of circle centre (0,0) and radius r $\{x^2 + y^2 = r^2\}$.

General equation of circle centre (-g,-f) and radius r $\{x^2 + y^2 + 2gx + 2fy + c = 0$, with $r = \sqrt{(g^2 + f^2 - c)}\}$.

*Equation of tangent at (x_1, y_1) to $x^2 + y^2 = r^2$.

Intersection of line and specific circle.

Trigonometric and algebraic parametric equations of circle (confined to verification that given parametric equations represent a circle).

Systems of circles *excluded*. Transformation techniques *excluded* from this section.

2. Plane vectors:

Addition, subtraction; multiplication by a scalar. Dot product. Unit ($\vec{i}$ and $\vec{j}$) vectors. For the vector $\vec{r} = x\vec{i} + y\vec{j}$, the related vector $\vec{r}^{\perp} = -y\vec{i} + x\vec{j}$.

3. Transformation geometry:

*Each transformation f of the plane M which has the coordinate form $(x,y) \rightarrow (x',y')$ where $x' = ax + by$, $y' = cx + dy$, and $ad-bc \neq 0$, maps each line to a line, each line segment to a line segment, each pair of parallel lines to a pair of parallel lines, and consequently each parallelogram to a parallelogram.

Proof confined to a specific transformation (numerical values for a, b, c and d).

Examples of the invariance or non-invariance of perpendicularity, distance, ratio of two distances, area, and ratio of two areas connected with specific parallelograms (including rectangles and squares) under transformations of the form:

$$x' = ax + by$$
$$y' = cx + dy$$

with numerical coefficients.

Use of matrix notation to be welcomed but not demanded.

Trigonometry

Trigonometry of the triangle: sine and cosine rules applied to the solution of triangles.

*Derivation of formulae 1 - 12 (see Appendix below). Application of formulae 1 - 20.

Solution of trigonometric equations, confined to equations such as $\sin \theta = 0$, $\cos \theta = 1/2$ (in both cases with all solutions required), $\cos 2\theta + \sin \theta = -1$ (which in one step can be converted to a quadratic equation) or ones such as $\sin \theta + \sin 3\theta = 0$ (which in one step can be converted to a product of terms equated to zero).

Radian measure of angles.

Use of the result $\lim_{x \to 0} (\sin x / x) = 1$.

Inverse functions $x \to \sin^{-1}x$ and $x \to \tan^{-1}x$ and their graphs.

Compositions excluded.

Periodicity excluded from this section.

Sequences and series

Sums of finite series of telescoping type such as $\sum_{n=1}^{m} \frac{1}{n(n+1)}$, arithmetic and geometric series, and $\sum_{n=1}^{m} n^2$.

$n!$, binomial coefficients $\binom{n}{r}$, $n \in N_0$; binomial series for positive integer exponent.

Questions on approximations will not be asked.

Informal treatment of limits of sequences; rules for sums, products, quotients; limits of sequences such as $\lim_{n \to \infty} \frac{n}{n+1}$, $\lim_{n \to \infty} r^n$ ($|r| < 1$).

Sums of infinite series of telescoping type, such as $\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$, $\sum_{n=0}^{\infty} x^n$, $\sum_{n=0}^{\infty} nx^n$ ($|x| < 1$).

Recurring decimals as infinite geometric series.

Series will be confined to those for which the sums are actually found. Limits will be found only for sequences given explicitly (not by recurrence relations). Tests for convergence excluded from core.

Functions and calculus

1. Functions:

Finding the period and range of a continuous periodic function, given its graph on scaled and labelled axis.

Range a closed interval $[a,b]$, $a, b \in Z$; period $\in N_0$. Periodic graph need not necessarily be trigonometric in type: e.g. saw-tooth graph.

Informal treatment of limits of functions; rules for sums, products and quotients.

2. Differential calculus:

*Derivations from first principles of x^2, x^3, $\sin x$, $\cos x$, $\sqrt{x}$, and $1/x$.

First derivatives of:

- polynomials, rational, power and trigonometric functions;
- $\tan^{-1}$, $\sin^{-1}$, exponential and logarithmic functions;
- *sums; *products; differences; *quotients; compositions of these.

*Proof by induction that $\frac{d}{dx}(x^n) = nx^{n-1}$.

Application to finding tangents to curves.

Simple second derivatives.

First derivatives of implicit and parametric functions.

Rates of change.

Maxima and minima.

} Problems involving modelling excluded. (See option).

Curve sketching of polynomials and of functions of form $\frac{a}{x + b}$ and $\frac{x}{x + b}$, with reference to turning points, points of inflection, and asymptotes.

Newton-Raphson method for finding approximate roots of cubic equations.

3. Integral calculus:

Integration techniques (integrals of sums, multiplying constants, and substitution) applied to:

- x^n
- $\sin nx$, $\cos nx$, $\sin^2 nx$, $\cos^2 nx$;
- e^{nx}
- functions of form: $\frac{1}{x+a}$, $\frac{1}{a^2+x^2}$, $\frac{1}{\sqrt{(a^2-x^2)}}$, $\sqrt{(a^2-x^2)}$.

Definite integrals with applications to areas and volumes of revolution (confined to cones and spheres).

Integration by parts and partial fractions <u>excluded</u>.

<u>Discrete mathematics and statistics</u>

1. Fundamental Principle of Counting: if one task can be accomplished in x different ways, and following this a second task can be accomplished in y different ways, then the first task followed by the second task can be accomplished in xy different ways.

   Permutations and combinations: concrete examples (examples with repetitions <u>excluded</u>).

2. Discrete probability: simple cases, with probability treated as relative frequency. For equally likely outcomes, probability = (number of outcomes of interest)/(number of possible outcomes). Examples including coin tossing, dice throwing, birthday distribution, card drawing, sex distribution, etc.

3. Statistics: calculation and interpretation of weighted mean and standard deviation.

4. Difference equations:
   *If a and b are the roots of the quadratic equation $px^2 + qx + r = 0$, and $s_n = la^n + mb^n$ for all n, then $ps_{n+2} + qs_{n+1} + rs_n = 0$ for all n.

   Examples of difference equations $ps_{n+2} + qs_{n+1} + rs_n = 0$ ($n >= 0$), (with s_0, s_1 given) to be solved, p, q, and r being specific numbers and the quadratic equation $px^2 + qx + r = 0$ having distinct roots.

OPTIONS
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At present, four optional topics are included;
- Further calculus and series;
- Further probability and statistics;
- Groups;
- Further geometry.

Further calculus and series

1. Maximum and minimum: applications to problems.

2. Integration by parts. No more than two steps (as for $e^x \sin x$ and $x^2 e^x$).

3. The ratio test confined to series of the form $\sum_{n=0}^{\infty} a_n x^n$.

4. nth derivatives; Maclaurin series for $(1 + x)^a$, e^x, $\log_e(1 + x)$, $\cos x$, $\sin x$, $\tan^{-1}x$.

5. Series expansion of π, using $\tan^{-1}x + \tan^{-1}a = \tan^{-1} \dfrac{x + a}{1 - xa}$

   $(|ax| < 1)$.

Further probability and statistics

1. Outcome space; events. Axioms of probability.

2. Addition of probabilities; conditional probability; independent events; multiplication of probabilities.

3. Distributions: binomial, normal.

4. Median and quartiles; mean and standard deviation and their limitations; choice of average.

5. Populations and samples. The sampling distribution of the mean. The role of the normal distribution. Standard error of a mean; confidence interval for a mean. Testing of null hypothesis at 5% level of significance.

Groups

1. Definition from axioms.

2. Examples (including commutative and

non-commutative, and finite and infinite, groups):

- groups of numbers under + and x, including finite and infinite groups from Z, Q, R, C, and Z_n (modulo arithmetic);
- matrix groups;
- Groups of permutations of degree up to 4;
- symmetry groups {for regular polygons with up to 6 sides (dihedral groups), non-square rectangle (Klein four-group), regular tetrahedron}.

3. Subgroups: examples in known groups; cyclic subgroups; centralizer of an element; centre of a group.

4. Isomorphism of groups.

5. Theorems:

I: Consequences of the axioms: uniqueness of identity and inverses; inverse of a product; cancellation (on left and on right); unique solution of equations ax=b and ya=b for x,y.

II: In any group G, if $g \in G$ then the set H = $\{g^n: n \in Z\}$ is a subgroup.

Use of Theorem II to classify all subgroups of a cyclic group.

III: If H, K are subgroups of G, then so also is $H \cap K$.

IV: Lagrange´s Theorem and the following consequences:
(a) any group of prime order is cyclic;
(b) the order of any element of a finite group G divides the order of G.

Proof of Lagrange´s Theorem <u>not</u> required.

V: If $\emptyset: G \rightarrow H$ is an isomorphism, then $\emptyset(e_G) = e_H$, and for any $x \in G$, $\emptyset(x)^{-1} = \emptyset(x^{-1})$.

VI: Any cyclic group of order n is isomorphic to the group of complex nth roots of unity; any infinite cyclic group is isomorphic to (Z,+).

<u>Further geometry</u>

1. Locus of harmonic conjugates of a point with respect to a circle. Focus-directrix definition of an ellipse; derivation of the equation of an ellipse in standard form.

2. Transformations f of the plane π which have the coordinate form $(x,y) \rightarrow (x',y')$ where:

$$x' = ax + by + k_1$$
$$y' = cx + dy + k_2$$

and $ad - bc \neq 0$. Use of matrices. Magnification ratio. Invariance of ratio of lengths on parallel lines, and of midpoints. Invariance of centroid of a triangle. Invariance of ratio of areas.

3. Deduction from results for a circle of similar results for an ellipse (dealing with the centre of an ellipse, tangents at the endpoints of a diameter of an ellipse, locus of midpoints of parallel chords of an ellipse, locus of harmonic conjugates of a point with respect to an ellipse (pole and polar), areas of all parallelograms circumscribed to an ellipse at the endpoints of conjugate diameters).

4. Similarity transformations, including enlargements and isometries. That similarity transformations map angles to equal angles, triangles to similar triangles, and circles to circles. Invariance under similarity transformations of orthocentre, incentre and circumcentre of a triangle.

APPENDIX: TRIGONOMETRIC FORMULAE

1. $\cos^2 A + \sin^2 A = 1$
2. Cosine formula: $a^2 = b^2 + c^2 - 2bc \cos A$
3. $\cos(A+B) = \cos A \cos B - \sin A \sin B$
4. $\cos 2A = \cos^2 A - \sin^2 A$
5. $\sin(A+B) = \sin A \cos B + \cos A \sin B$
6. $\sin 2A = 2 \sin A \cos A$
7. $\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$
8. $\tan 2A = \dfrac{2 \tan A}{1 - \tan^2 A}$
9. $\cos 2A = \dfrac{1 - \tan^2 A}{1 + \tan^2 A}$
10. $\sin 2A = \dfrac{2 \tan A}{1 + \tan^2 A}$
11. $\cos^2 A = (1 + \cos 2A)/2$
12. $\sin^2 A = (1 - \cos 2A)/2$
13. $2 \cos A \cos B = \cos(A+B) + \cos(A-B)$
14. $2 \sin A \cos B = \sin(A+B) + \sin(A-B)$

15. $2 \sin A \sin B = \cos(A-B) - \cos(A+B)$

16. $2 \cos A \sin B = \sin(A+B) - \sin(A-B)$

17. $\cos A + \cos B = 2 \cos \frac{A+B}{2} \cos \frac{A-B}{2}$

18. $\cos A - \cos B = -2 \sin \frac{A+B}{2} \sin \frac{A-B}{2}$

19. $\sin A + \sin B = 2 \sin \frac{A+B}{2} \cos \frac{A-B}{2}$

20. $\sin A - \sin B = 2 \cos \frac{A+B}{2} \sin \frac{A-B}{2}$

Note:
These formulae appear on p. 9 of the Mathematical Tables Approved for use at the Public Examinations conducted by the Department of Education and the Civil Service Commissioners (Dublin: the Stationery Office, n. d.). It should be noted that the formula on the last line of that page, $e^{in\theta} = (\cos\theta + i \sin\theta)^n = \cos n\theta + i \sin n\theta$, is excluded from the course.

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3. ORDINARY LEVEL

3.1 INTRODUCTION

Two courses are offered at Ordinary level: Ordinary, and Ordinary Alternative. The Ordinary Alternative course (which was introduced in 1990) is currently sanctioned for examination up to and including Summer 1994.

3.2 ORDINARY COURSE: RATIONALE, STYLE AND AIMS

For many of the students for whom the Ordinary course was designed, mathematics is essentially a service subject -- providing knowledge and techniques that will be needed in future for their study of scientific, economic, business and technical subjects. For other Ordinary course students, however, the Leaving Certificate may provide their last formal encounter with mathematics. In general, therefore, the course should equip students who will use mathematics in further study for the courses that they will pursue; but it should also cater suitably for students who will not proceed to further study of mathematics or related subjects. Hence, material is chosen for its intrinsic interest and general applicability as well as its provision of techniques useful in further education.

Students who follow the Ordinary course may have had fairly limited prior contact with abstract mathematics. The course therefore moves gradually from the relatively concrete and practical to more abstract and general concepts. As well as equipping the students with important tools, it offers opportunities for them to deepen their understanding and appreciation of mathematics and to experience some of its classical "powerful ideas".

For the target group, particular emphasis can be given to aims concerned with the use of mathematics. Due attention should be given to maintenance of the more basic skills, especially in applications of arithmetic and algebra (where students' shortcomings have been seen to stand in the way of their own progress).

3.3 ORDINARY COURSE: PRIOR KNOWLEDGE

Knowledge of the content of the Junior Certificate Ordinary course will be assumed.

3.4 ORDINARY COURSE: ASSESSMENT OBJECTIVES

The assessment objectives are the fundamental objectives A, B, C, D and E (see Section 1.3), interpreted in the context of the following aims of the Ordinary course:

- a widening of the span of the students' understanding from a relatively concrete and practical level to a more abstract and general one;
- the acquisition of mathematical techniques and their use in context;
- proficiency in basic skills of arithmetic and algebra.

Notes:

1. It is desirable that students following the course would make intelligent and proficient use of calculators (and would carry their

expertise into their lives beyond school); and it is envisaged that calculators would normally be used as a tool during the teaching, learning and examining of the course. Unlike the situation for the Ordinary Alternative syllabus, however, the course has not been specifically designed around use of machines, and assessment of calculator skills is not a "core" or essential requirement.

2. As indicated by objective E, the students should present their work comprehensibly; this is especially relevant when they are using calculators.

3.5 <u>ORDINARY COURSE: STRUCTURE AND CONTENT</u>

The syllabus is presented in the form of a <u>core</u> and a set of <u>options</u>. It is envisaged that students would study the whole of the core and one option.

CORE

Arithmetic

1. The operations of addition, subtraction, multiplication and division of rational numbers, and the relations < and >, applied to practical problems involving counting and measuremant (units {S.I. where appropriate} of length, area, volume, time, mass, temperature, and money); averages; rates; proportion; percentages; money transactions, including compound interest; taxation.	Problems may involve use of variables and knowledge of proportionality. Taxation problems may include those working from tax to original pay etc.
2. Estimation and approximation.	
Relative error: definition (relative error = |error/true value|, or -- if the true value is unavailable -- |error/estimate|); percentage error. Accumulation of error by addition or subtraction only.	Intelligent and accurate use of calculator required.
Simpson´s method for approximating to areas of irregular figures.	
Use of calculators and/or tables; scientific notation.	
3. Powers and nth roots (for example as used in compound interest formulae).	
4. Areas: triangles, discs, sectors of discs; figures made from combinations of these. Volumes: sphere, hemisphere, right cone, right prism, rectangular solids; solids made from combinations of these.	

Algebra

1. Manipulation of formulae including simple algebraic fractions.

   Laws of indices: $x^a x^b = x^{a+b}$; $(xy)^a = x^a y^a$; $(x^a)^b = x^{ab}$.

   Use of fractional and negative indices, e.g. $(-8)^{2/3}$, $(1/4)^{-1/2}$. Solution of equations such as $5^x = 1/25$.

   Solution of quadratic equations with rational coefficients.

   The Factor Theorem for polynomials of degree two or three.

   Factorisation of such polynomials (the linear and quadratic factors having integer coefficients).

2. Unique solution of simultaneous linear equations with two unknowns.

   Solution of one linear and one quadratic equation with two unknowns (e.g. $2x - y = 1$, $x^2 + y^2 = 9$).

3. Inequalities: solution of inequalities of the form $g(x) < k$, where $g(x) = ax + b$, and $a, b, k \in Q$.

4. Complex numbers: Argand diagram, modulus, complex conjugate.

   Addition, subtraction, multiplication, division.

Geometry

1. Synthetic geometry:

   Theorems (to be proved): "Cuts" will <u>not</u> be asked (see Option: Further Geometry).

   I: The sum of the degree-measures of the angles of a triangle is 180°.

   Corollary I: The degree-measure of an exterior angle of a triangle is equal to the degree-measure of the sum of the two remote interior angles.

   Corollary II: An exterior angle of a triangle is greater than either remote interior angle.

II: Opposite sides of a parallelogram have equal lengths.

III: If three parallel lines make intercepts of equal length on a transversal, then they will also make intercepts of equal length on any other transversal.

IV: A line which is parallel to one side-line of a triangle, and cuts a second side, will cut the third side in the same proportion as the second.

V: If the three angles of one triangle have degree-measures equal, respectively, to the degree-measures of the angles of a second triangle, then the lengths of the coresponding sides of the two triangles are proportional.

VI (Pythagoras): In a right-angled triangle, the square of the length of the side opposite to the right-angle is equal to the sum of the squares of the lengths of the other two sides.

VII (Converse of Pythagoras' Theorem): If the square of the length of one side of a triangle is equal to the sum of the squares of the lengths of the other two sides, then the triangle has a right-angle and this is opposite the longest side.

VIII: The products of the lengths of the sides of a triangle by the corresponding altitudes are equal.

IX: If the lengths of two sides of a triangle are unequal, then the degree-measures of the angles opposite to them are unequal, with the greater angle opposite to the longer side.

X: The sum of the lengths of any two sides of a triangle is greater than that of the third side.

2. Coordinate geometry:

Coordinates; distance between points; area of triangle; midpoint of line segment; slope.

Line:
- equation of line in the forms $y = mx + c$ and $y - y_1 = m(x - x_1)$;
- line through two given points;
- lines parallel to and lines perpendicular to a given line and through a given point;
- intersection of two lines.

Circle:
- the equation $x^2 + y^2 = a^2$;
- intersection of a line and a circle;) restricted to a circle
- proving a line is a tangent to a circle;) centre the origin.
- equation of circle in the form $(x - h)^2 + (y - k)^2 = a^2$. Given equation, obtain centre; and vice versa.

3. Enlargements:

Enlargement of a rectilinear figure by the ray method. Centre of enlargement. Scale factor k. Two cases to be considered:
- $k>1$, $k \in Q$ (enlargement);
- $0<k<1$, $k \in Q$ (reduction).

A triangle abc with centre of enlargement a, enlarged by a scale factor k, gives an image triangle ab′c′ with bc parallel to b′c′.

Object length, image length. calculation of scale factor.

Finding the centre of enlargement.

A region when enlarged by a scale factor k has its area multiplied by a factor k^2.

Trigonometry

Trigonometry of triangle; area of triangle; use of sine and cosine rules. Proofs not required.

Definitions of sin x and cos x for all values of x. Definition of tan x.

Area of sector of circle; length of arc.

Formulae for sin $(A \pm B)$ and cos $(A \pm B)$. Proofs not required

Finite sequences and series

Informal treatment of sequences.

Arithmetic and geometric sequences.

Sum to n terms of arithmetic and geometric series.

Functions and calculus

1. Functions:

A function as a set of couples, no two couples having the same first element; that is, a function as a particular form of See Relations section of Junior Certificate Foundation course. This association between the elements of two sets.

aspect not to be examined.

Function considered as specified by a formula (or rule or procedure or curve) which establishes such an association by consistently transforming input into output. Examples of functions; examples which are not functions.

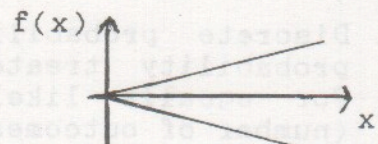
Examples:

(i) f: $x \rightarrow \begin{cases} 1, & x \text{ even} \\ 0, & x \text{ odd}, \end{cases}$ $x \in N$.

(ii) alphanumeric set $\rightarrow$ ASCII code or bar code.

(iii) f: $x \rightarrow 2x$, $x \in R$.

Counter example:



Use of function notation:

$f(x) =$

$f\colon\ x \rightarrow$

$y =$

Graphs of functions f of linear, quadratic and cubic type and of $\frac{1}{x + a}$. Use of graphs to find approximate solutions to inequalities $f(x) <= b$ and to equalities $f(x) = cx + d$.

Finding the period and range of a continuous periodic function, given its graph on scaled and labelled axis.

Range a closed interval $[a,b]$, $a, b \in Z$; period $\in$ No. Periodic graph need not necessarily be trigonometric in type: e.g. saw-tooth graph.

2. Calculus:

Informal treatment of limits of functions.

Derivations from first principles of polynomials of degree <= 2

First derivatives of:
- polynomials and rational functions;
- sums, products, differences, quotients.

Easy applications of the chain rule.

Rates of change, e.g. speed, acceleration. Tangents.

Calculation of maxima and minima of quadratic and cubic functions.

Discrete mathematics and statistics

1. Fundamental Principle of Counting: if one task can be accomplished in x different ways, and following this a second task can be accomplished in y different ways, then the first task followed by the second task can be accomplished in xy different ways.

   Permutations and combinations: concrete examples (examples with repetitions excluded).

2. Discrete probability: simple cases, with probability treated as relative frequency. For equally likely outcomes, probability = (number of outcomes of interest)/(number of possible outcomes). Examples including coin tossing, dice throwing, birthday distribution, card drawing (one or two cards), sex distribution, etc.

3. Statistics: histogram; cumulative frequency; ogive, median, interquartile range; approximate mean of a grouped frequency distribution; weighted mean; concept of dispersion; standard deviation.

   Median obtained from array or cumulative frequency graph; finding median from histogram excluded.

OPTIONS
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At present, four optional topics are included:
- Further geometry;
- Plane vectors;
- Further sequences and series;
- Linear programming.

Further geometry

Theorems (to be proved):

"Cuts" on these theorems may be asked. (Construction lines will not be required.)

A: The degree-measure of an angle subtended at a centre of the circle by a chord is equal to twice the degree-measure of any angle subtended by the chord at a point of the arc of the circle which is on the same side of the chordal line as is the centre.

Corollary: The angle which are subtended by a chord of the circle at points of the circle which are on one side of the chordal line have equal degree-measures.

B: A line is a tangent to a circle at a point t of the circle if and only if it passes through t and is perpendicular to the line through t and the centre.

C: If [ab] and [cd] are chords of a circle and the lines ab and cd meet at the point k, than $|ak|\ |kb| = |ck|\ |kd|$.
If [ab] is a chord of a circle, $k \in ab$ and kt is a tangent to the circle at the point t, then $|ak|\ |kb| = |kt|^2$.

D: An angle between a tangent ak and a chord [ab] of a circle has degree-measure equal to that of any angle in the alternate segment.

Plane vectors

Addition, subtraction; multiplication by a scalar. Dot product.

Unit ($\vec{i}$ and $\vec{j}$) vectors. For the vector $\vec{r} = x\vec{i} + y\vec{j}$, the related vector $\vec{r}^{\perp} = -y\vec{i} + x\vec{j}$.

Further sequences and series

Applications of finite arithmetic and geometric series.

Informal treatment of limits of sequences, especially
$\lim_{n \to \infty} r^n$, $|r| < 1$.

Sum of infinite geometric series. Application to infinite decimals

Binomial expansion, confined to $(1 + x)^n$, $(1 - x)^n$, $n \in N$, $n <= 7$.

Linear programming

Graphing the solution set of linear inequalities in two variables.

Elementary linear programming, limited to a region bounded by two lines and the two axes.

3.6 ORDINARY ALTERNATIVE COURSE: RATIONALE, STYLE AND AIMS

The Ordinary Alternative course is intended primarily as a terminal course in mathematics, equipping students with the knowledge and techniques required in everyday life and in various kinds of employment. It also lays the groundwork for students who may proceed to further training, where relevant. It should therefore provide students with the mathematical tools needed in their daily life and work; but it should do so in a context designed to build the students' confidence, their enjoyment of mathematics, and their recognition of its role in the world about them. Hence, material is chosen for its intrinsic interest and immediate applicability as well as its usefulness in life beyond school.

The course is designed for students who have had only very limited acquaintance with abstract mathematics. Basic knowledge is maintained and enhanced by being approached in an exploratory and reflective manner -- availing of students' increasing maturity -- rather than by simply repeating work done in the Junior Cycle. Concreteness is provided in particular by extensive use of the calculator. This serves as an investigative tool as well as an object of study and a readily available resource. Abstract concepts are introduced constructively via a multiplicity of carefully graded examples.

For the target group, particular emphasis can be given to aims concerned with the use of mathematics in everyday life and work -- especially as regards intelligent and proficient use of calculators -- and with the recognition of mathematics in the environment.

3.7 ORDINARY ALTERNATIVE COURSE: PRIOR KNOWLEDGE

Knowledge of the content of the Junior Certificate Foundation Course will be assumed.

3.8 ORDINARY ALTERNATIVE COURSE: ASSESSMENT OBJECTIVES

The assessment objectives are the fundamental objectives A, B, C, D and E (see Section 1.3), interpreted in the context of the following aims of the Ordinary Alternative course:

- development of students' understanding of mathematical knowledge and techniques required in everyday life and employment;
- acquisition of mathematical knowledge that is of immediate applicability and usefulness;
- introduction of the students to a limited idea of the mathematically abstract;
- maintenance and enhancement of students' basic mathematical knowledge and skills;
- encouragement of accurate and efficient use of the calculator;
- promotion of students' confidence in working with mathematics.

Notes:

1. The accurate and efficient use of the calculator is a specific assessment objective.

2. As indicated by objective E, the students should present their work comprehensibly; this is especially relevant when they are using calculators.

3.9 <u>ORDINARY ALTERNATIVE COURSE: STRUCTURE AND CONTENT</u>

At present the syllabus is presented without options. It is therefore envisaged that students would study the entire course.

CONTENT
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<u>Number systems</u>

Revision of the following, using the calculator for all relevant aspects, in line with the principles laid down above:

1. Development of the systems N of natural numbers, Z of integers, Q of rational numbers, and R of real numbers. The operations of addition, multiplication, subtraction and division. Representation of numbers on a line. Inequalities. Decimals. Powers and roots. Scientific notation.

2. Factors, multiples, prime numbers in N. Prime factor theorem.

3. Use of brackets Conventions as to the order of precedence of operations. Easy algebraic manipulations leading to the handling of formulae and solution of equations.

<u>Arithmetic</u>

Use of calculator for all relevant operations in the following:

1. Approximation and error; rounding off. Relative error, percentage error, tolerance Very large and very small numbers on the calculator. Limits to accuracy of calculators.

2. Substitution in formulae. — Main stages of calculation should be shown.

3. Proportion. Percentage. Averages. Average rates of change (with respect to time).

4. Compound interest formulae:

$$A = P\left(1 + \frac{r}{100}\right)^n,$$

$$P = A \Big/ \left(1 + \frac{r}{100}\right)^n$$

Both formulae provided in examinations; n a natural number. Applications to such areas as depreciation included.

5. VAT. Rates. Income tax (including PRSI); emergency tax; tax tables.

6. Domestic finance and household management: preparing a domestic budget; ESB / Gas / Telecom bills.
Social welfare allowances etc.; money-lending. Implications of <u>change</u> in ESB and other charges for the domestic budget.

7. Currency transactions, including commission.
Use of reciprocal key (where available) on calculator.

8. Costing. Materials and labour. Wastage.

9. Metric system. Change of units. Everyday Imperial units.
Conversion factors provided for Imperial units.

<u>Areas and volumes</u>

Use of calculator for all relevant operations in the following:

1. Plane figures: disc, triangle, rectangle, square, hollow rectangle, H-figure, parallelogram, trapezium.
Solid figures: right cone, rectangular block, cylinder, sphere, right prism.
See Appendix. Questions will be confined to the variables given in the formulae ("engineer's handbook" approach).

2. Use of Simpson's Rule to approximate area.

<u>Algebra</u>

Consideration of the following, using calculator where relevant:

1. $x + a = b$
2. $ax = b$ } $a, b, \in Q$
3. $ax + b = c$
4. $ax + b = cx$ } $a, b, c, d \in Z$
5. $ax + b = cx + d$
6. $ax + by = c$, $dx + ey = f$ } $a, b, c, d, e, f \in Z$

a and b may be finite decimals.

Examination restricted to case of unique solutions.

Problems giving rise to equations of type 1 - 6.

7. $x^2 = a, \ a \in Q^+$
8. $x^2 + a = b, \ b - a > 0, \ a, b \in Q$

9. $ax^2 = b$, $a, b \in Q^+$	
10. $ax^2 + b = c$, $(c-b)/a > 0$, $a, b, c \in Z$.	
11. $ax^2 + bx + c = 0$, $b^2 >= 4ac$, $a, b, c \in Z$.	Use of formula (provided in examinations).

<u>Statistics and probability</u>

1. Fundamental Principle of Counting: if one task can be accomplished in x different ways, and following this a second task can be accomplished in y different ways, then the first task followed by the second task can be accomplished in xy different ways.	Use in examples.
2. Discrete probability: simple cases, with probability treated as relative frequency. For equally likely outcomes, probability = (number of outcomes of interest)/(number of possible outcomes). Examples including coin tossing, dice throwing, birthday distribution, card drawing (one or two cards), sex distribution, etc.	
3. Statistics: graphical and tabular representation of statistical data; grouped and ungrouped frequency distributions. Mean; cumulative frequencies and cumulative frequency graph; median; weighted mean. Concept of dispersion; standard deviation of ungrouped array of not more than ten numbers.	Emphasis on use of calculator. Median obtained from array or cumulative frequency graph; finding median from histogram <u>excluded</u>.

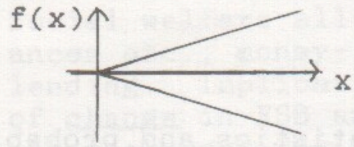
<u>Trigonometry</u>

Sine, cosine and tangent as ratios in a right-angled triangle.	
Solving for one unknown in a right-angled triangle.	Problems which give rise to this included.

<u>Functions and graphs</u>

1. A function as a set of couples, no two couples having the same first element; that is, a function as a particular form of association between the elements of two sets.	See Relations section of Junior Certificate Foundation course. This aspect not to be examined.
Function considered as specified by a formula (or rule or procedure or curve) which establishes such an association by consistently transforming input into output. Examples of functions; examples which are not functions.	Examples: (i) f: x→ { 1, x even; 0, x odd, x ∈ N. (ii) alphanumeric set → ASCII code or bar code. (iii) f: $x \rightarrow 2x$, $x \in R$.

Counter example:



2. Study of the following functions in detail:

   f: x→mx; m ∈ Q, x ∈ R. Effect on the graph of varying m. Solution of f(x) = 0.

   Emphasis on fact that graph passes through origin.

   f: x→mx + c; m, c ∈ Q, x ∈ R. Significance of c. Solution of f(x) = 0.

   Number of points needed to draw these graphs.

   f: x→x^2; x ∈ R. Estimation of $\sqrt{2}$; significance of two solutions of f(x) = 2.

   f: x→x^2 + c; c, x ∈ R. Effect on the graph of varying c. Solution of f(x) = 0, c < 0.

   f: x→ ax^2; a, x ∈ R. Effect on the graph of varying a.

   f: x→ ax^2 + bx + c; values of x for which f(x) is maximum / minimum. Intervals of x for which f(x) is increasing / decreasing. Approximation to (real) solutions of f(x) = d.

3. Experimental results. Fitting a straight line to a set of experimental data. Prediction.

4. Finding the period and range of a continuous periodic function, given its graph on scaled and labelled axis.

   Range a closed interval [a,b], a, b ∈ Z; period ∈ No. Periodic graph need not necessarily be trigonometric in type: e.g. saw-tooth graph. Radian measure <u>excluded</u>.

5. Interpretation of graphs in the following cases;

   Case 1:
   cases in which information is available only at plotted points

   Examples:
   - currency fluctuations
   - inflation
   - employment / unemployment
   - temperature
   - temperature chart

(medical)
- pollen count
- lead levels
- smog

Case 2:
restricted to the following:
- distance/time
- speed/time
- depth of liquid/time
- conversion of units

Case 2: graphs are continuous.

Interpretation to cover: given value of one variable, estimating from graph the corresponding value of the other; significance of changes.

Geometry

1. Co-ordinate geometry:

Formulae will be given in examinations.

Distance between two points.

Slope of a line through two points. Parallel lines. Perpendicular lines.

Midpoint of a line segment.

Equation of line: $y = mx + c$. Obtaining equation of line, given slope and one point or given two points.

2. Geometrical results: knowledge of the following and use in numerical examples:
   (a) Vertically opposite angles are equal;
   (b) When a transversal cuts two lines, the corresponding angles are equal, and the alternate angles are equal;
   (c) Opposite sides and angles of a parallelogram are equal;
   (d) The sum of the angles of a triangle is 180°;
   (e) The base angles of an isosceles triangle are equal;
   (f) The angle on a (straight) line is 180°;
   (g) The Theorem of Pythagoras;
   (h) The angle in a semicircle is a right angle.

Proofs <u>excluded</u>.

"Equal" means equal in measure.

3. Constructions:
   (a) To draw a perpendicular from a given point on a line;
   (b) To draw a perpendicular from a point at the end of a line segment;
   (c) To draw a perpendicular from a point not on a line to a given line;
   (d) To construct an angle of 60°;

(e) To construct an angle equal to a given angle;
(f) To draw a line parallel to a given line through a point;
(g) To construct a parallelogram (given sufficient data);
(h) To draw the circumscribed circle of a given triangle;
(i) To draw the inscribed circle of a given triangle;
(j) To draw the tangent to a circle at a given point on the circle.

4. Enlargements:

Enlargement of a rectilinear figure by the ray method. Centre of enlargement. Scale factor k. Two cases to be considered:
- $k>1$, $k \in Q$ (enlargement);
- $0<k<1$, $k \in Q$ (reduction).

A triangle abc with centre of enlargement a, enlarged by a scale factor k, gives an image triangle ab′c′ with bc parallel to b′c′.

Object length, image length. calculation of scale factor.

Finding the centre of enlargement.

A region when enlarged by a scale factor k has its area multiplied by a factor k^2.

5. Nets of rectangular solids, pyramids, and right prisms with triangular cross-section.

6. Repeating patterns. Identification of axial symmetry, planes of symmetry, central symmetry, and rotational symmetry in given figures.	Co-ordinate treatment excluded. Assessment limited to identification of symmetries as indicated and of units that repeat.

Investigations

Development of concepts and strategies for investigating mathematical problems and observing general patterns and results.

APPENDIX: "ENGINEER′S HANDBOOK"

The handbook is intended to be used as follows:

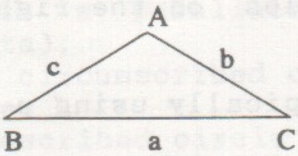
- students select the formula with the required unknown on the left-hand side and with values available for all variables on the right-hand side;

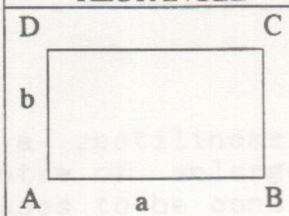
- they substitute values for variables on the right-hand side of the formula;

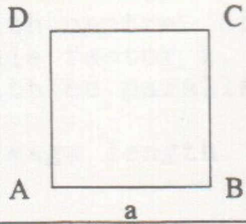
- they evaluate the required answer, typically using a calculator.

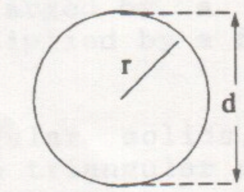
This obviates algebraic manipulation. It also provides experience of an approach widely used in practical applications. (See following page.)

LENGTH

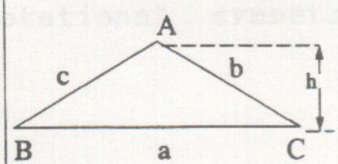
TRIANGLE	LENGTH (L)	FORMULAE
	$L = a + b + c$	$a = L - b - c$ $b = L - a - c$ $c = L - a - b$

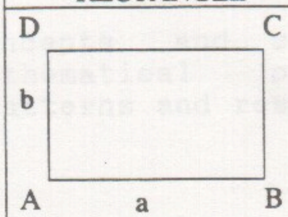
RECTANGLE	LENGTH (L)	FORMULAE
	$L = 2(a+b) = 2a + 2b$	$a = \frac{(L-2b)}{2}$ $b = \frac{(L-2a)}{2}$

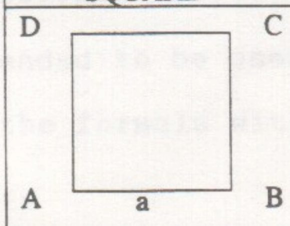
SQUARE	LENGTH (L)	FORMULAE
	$L = 4a$	$a = \frac{L}{4}$

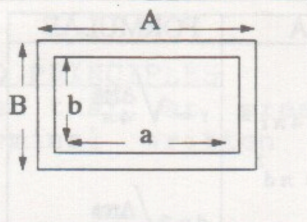
CIRCLE	LENGTH (L)	FORMULAE
	$L = 2\pi r$ $L = \pi d$	$d = 2r$, $r = \frac{d}{2}$ $r = \frac{L}{2\pi}$ $d = \frac{L}{\pi}$

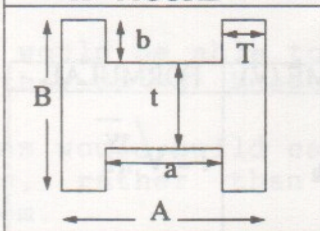
AREA

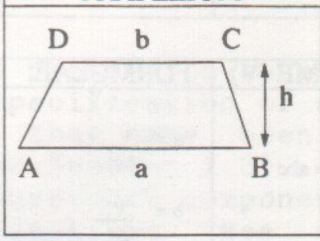
TRIANGLE	AREA	FORMULAE
	$Area = \frac{ah}{2}$	$a = \frac{2(area)}{h}$ $h = \frac{2(Area)}{a}$

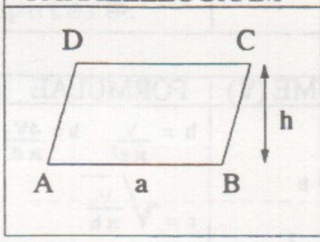
RECTANGLE	AREA	FORMULAE
	$Area = ab$	$a = \frac{Area}{b}$ $b = \frac{Area}{a}$

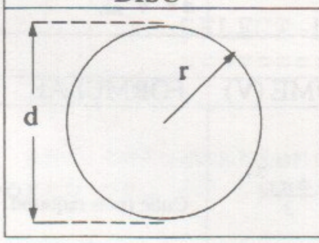
SQUARE	AREA	FORMULAE
	$Area = a^2$	$a = \sqrt{Area}$

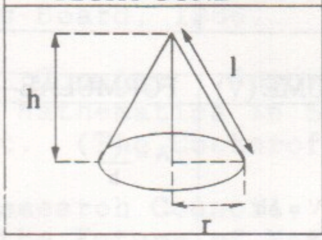
HOLLOW RECTANGLE	AREA	FORMULAE
	$\text{Area} = AB - ab$	$A = \dfrac{(\text{Area} + ab)}{B}$ $B = \dfrac{(\text{Area} + ab)}{A}$ $a = \dfrac{(AB - \text{Area})}{b}$ $b = \dfrac{(AB - \text{Area})}{a}$

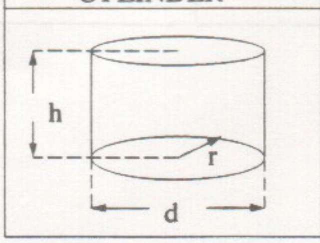
H - FIGURE	AREA	FORMULAE
	$\text{Area} = AB - 2ab$ $\text{Area} = at + 2BT$ Note: $A = a + 2T$ $B = 2b + t$	$A = \dfrac{(\text{Area} + 2ab)}{B}$ $B = \dfrac{(\text{Area} + 2ab)}{A}$ $a = \dfrac{(AB - \text{Area})}{2b}$ $b = \dfrac{(AB - \text{Area})}{2a}$

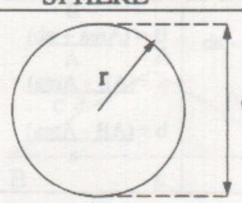
TRAPEZIUM	AREA	FORMULAE
	$\text{Area} = \dfrac{h(a + b)}{2}$	$a = \dfrac{2(\text{Area})}{h} - b$ $b = \dfrac{2(\text{Area})}{h} - a$ $h = \dfrac{2(\text{Area})}{(a + b)}$

PARALLELOGRAM	AREA	FORMULAE
	$\text{Area} = ah$	$a = \dfrac{\text{Area}}{h}$ $h = \dfrac{\text{Area}}{a}$

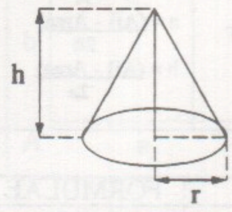
DISC	AREA	FORMULAE
	$\text{Area} = \pi r^2$ $\text{Area} = \dfrac{\pi d^2}{4}$	$r = \sqrt{\dfrac{\text{Area}}{\pi}}$ $d = \sqrt{\dfrac{4(\text{Area})}{\pi}}$

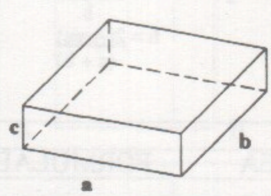
RIGHT CONE	AREA	FORMULAE
	$\text{Area} = \pi r l$ Note: $l^2 = r^2 + h^2$	$r = \dfrac{\text{Area}}{\pi l}$ $l = \dfrac{\text{Area}}{\pi r}$

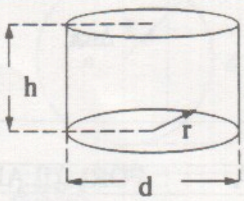
CYLINDER	AREA	FORMULAE
	$\text{Area} = 2 \pi r h$ $\text{Area} = \pi d h$	$r = \dfrac{\text{Area}}{2\pi h}$ $h = \dfrac{\text{Area}}{2\pi r}$ $d = \dfrac{\text{Area}}{\pi h}$ $h = \dfrac{\text{Area}}{\pi d}$

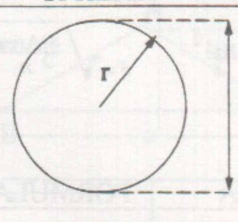
SPHERE	AREA	FORMULAE
	Area = $4 \pi r^2$ Area = πd^2	$r = \sqrt{\frac{Area}{4\pi}}$ $d = \sqrt{\frac{Area}{\pi}}$

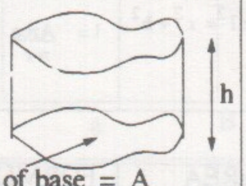
VOLUME

RIGHT CONE	VOLUME (V)	FORMULAE
	$V = \frac{\pi r^2 h}{3}$	$r = \sqrt{\frac{3V}{\pi h}}$ $h = \frac{3V}{\pi r^2}$

RECTANGULAR BLOCK	VOLUME (V)	FORMULAE
	V = abc	$a = \frac{V}{bc}$ $b = \frac{V}{ac}$ $c = \frac{V}{ab}$

CYLINDER	VOLUME (V)	FORMULAE
	$V = \pi r^2 h$ $V = \frac{\pi h d^2}{4}$	$h = \frac{V}{\pi r^2}$ $h = \frac{4V_2}{\pi d}$ $r = \sqrt{\frac{V}{\pi h}}$ $d = \sqrt{\frac{4V}{\pi h}}$

SPHERE	VOLUME (V)	FORMULAE
	$V = \frac{4 \pi r^3}{3}$ $V = \frac{\pi d^3}{6}$	Cube roots required

RIGHT PRISM	VOLUME (V)	FORMULAE
 Area of base = A	V = Ab	$A = \frac{V}{h}$ $h = \frac{V}{A}$

4. ASSESSMENT

4.1 STRUCTURES AND PRINCIPLES

It is envisaged that, at present, the courses would be assessed by means of terminal written examinations. The following principles would apply:

A. The status and standing of the Leaving Certificate would be maintained.

B. Candidates would be able to demonstrate what they do know rather than what they do not know.

C. Examinations would build candidates´ confidence that they can do mathematics, rather than undermining the confidence of those who attempt them.

4.2 NOTE

Restriction at present to assessment by formal written examination has governed the specification of assessment objectives (see Sections 2.4, 3.4 and 3.8); they have been limited to a subset of the fundamental objectives (see Section 1.3). In the future, it may be possible to introduce a coursework component. This would facilitate assessment of the other objectives (see section 1.3), notably problem-solving, communicative and creative skills, and in particular of work done with the aid of computers.

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5. SELECT BILIOGRAPHY

Among the national and international literature consulted, the following three national reports are of particular relevance:

1. Curriculum and Examinations Board. Mathematics Education: Primary and Junior Cycle Post-Primary. Dublin: Curriculum and Examinations Board, 1986.

2. Mathematics Counts. Report of the Committee of Inquiry into the Teaching of Mathematics in Schools under the Chairmanship of Dr. W. H. Cockcroft. (The Cockcroft Report.) London: HMSO, 1982.

3. National Research Council. Everybody Counts: a Report to the Nation on the Future of Mathematics Education. Washington, D. C.: National Academy Press, 1989.

THE NATIONAL COUNCIL FOR CURRICULUM AND ASSESSMENT

Procedures for drawing up National Syllabuses

Course Committees established by the NCCA are responsible for drawing up syllabuses and associated guidelines for subjects at post-primary level. Senior Cycle (Leaving Certificate) Course Committees have the following membership:

Association of Secondary Teachers, Ireland:	2 members
Teachers Union of Ireland:	2 members
Joint Managerial Body for Secondary Schools:	1 member
Association of Community and Comprehensive Schools:	1 member
Subject Association:	1 member
Irish Vocational Education Association:	1 member
National Council for Educational Awards:	1 member
Committee of Heads of Universities:	2 members
Department of Education (Inspectorate):	2 members

Recommendations of Course Committees are submitted to the NCCA for approval. The NCCA having considered such recommendations, submits the syllabus to the Minister for Education, whose responsibility it is to approve and issue it to schools.