LEAVING CERTIFICATE EXAMINATION, 1994

MATHEMATICS — ORDINARY LEVEL

SAMPLE PAPER I (300 marks) - 2½ hours

Attempt **SIX QUESTIONS** (50 marks each)

**Marks may be lost if all your work is not clearly shown
or if you do not indicate where a calculator has been used.**

1. (a) IR£19·50 pocket money is divided between two pupils in the ratio 8:5. How much does each get?

   (b) Calculate the percentage error in taking $50 + 60$ as an approximation for $52·47 + 64·87$.

   (c) A person has a tax-free allowance of IR£2400 per annum. The annual take home pay is IR£9268.
   Calculate the annual gross pay if tax is paid at 32%.

2. (a) Solve

$$3x - 2y = -10$$
$$2x + 3y = 2$$

   (b) Find the solution set E of $5x - 6 < 4$.

   Find the solution set H of $-4x \leq 3$.

   Find $E \cap H$.

   (c) Write $\dfrac{125}{\sqrt{5}}$ as a power of 5, and solve for x the equation
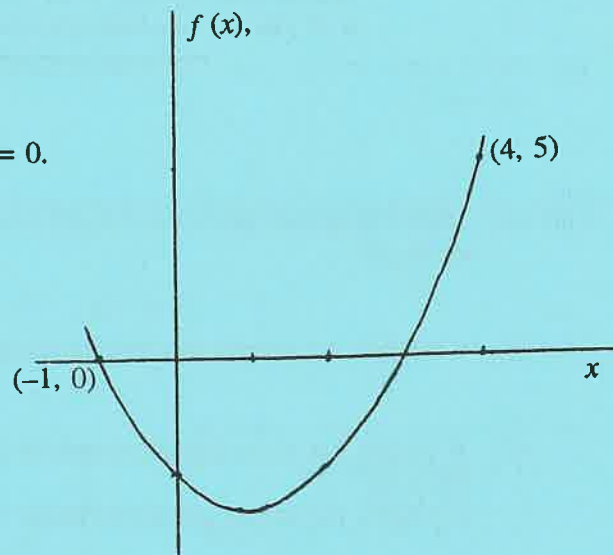
$$5^{2x+1} = \left(\frac{125}{\sqrt{5}}\right)^3.$$

3. (a) Express t in terms of p and q when

$$p + \frac{t}{q} = 1, \quad q \neq 0.$$

(b) Show that $x = 4$ is a root of $x^3 - 2x^2 - 11x + 12 = 0$, and find the other two roots.

(c) The graph of the quadratic function $x \to f\ (x), x \in R$, is as shown. Express $f\ (x)$ in the form $x^2 + bx + c$.

$g\ (x) = px + q$, where $g\ (0) = -3$ and $g\ (1) = 0$. Find the value of p and the value of q. Find the values of x for which $g\ (x) = f\ (x)$.

$f\ (x)$,

$(4, 5)$

$(-1, 0)$

x

4. (a) Let $z_1 = 2 + 3i$ and $z_2 = 4 + i$, where $i^2 = -1$.

Plot

   (i) z_1
   (ii) z_2
   (iii) $z_2 - z_1$

on the Argand diagram.

(b) Express $\dfrac{1}{2 + 3i}$ in the form $p + qi, \quad p, q \in R$.

(c) (i) Solve for real x and real y
$$2x + (x + y)\ i = 4 - 5i.$$

   (ii) Let $z = 3 - 2i$.
   Solve for real s and real t

$$\frac{s + it}{1 + 2i} = \bar{z},$$

where $\bar{z}$ denotes the complex conjugate of z.

5. **(a)** The first term, T_1, of an arithmetic sequence is 9 and the common difference is 4.

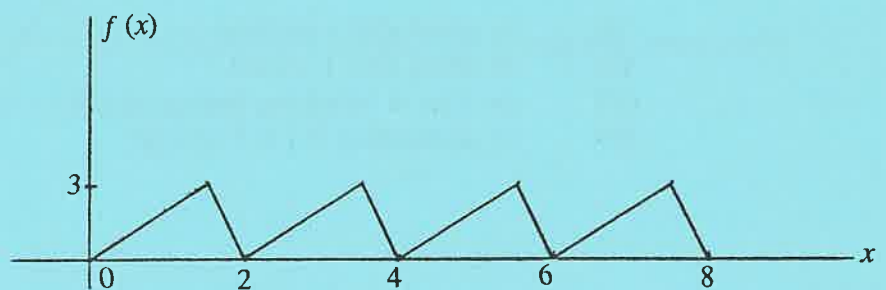Find T_5, the fifth term and T_n, the n.th. term.

(b) In an arithmetic sequence, S_n, the sum to n. terms, is

$$S_n = 5n - 2n^2.$$

Find T_1 and the common difference.

(c) What sum of money would amount to IR£8843·18 in 4 years at 8% per annum compound interest ?

6. **(a)** The graph shows portion of a periodic function $f : x \rightarrow f(x)$.
Write down the range and period of the function.

(b) The formula for relating degrees fahrenheit, F, to degrees celcius, C, is

$$C = \frac{5}{9} (F - 32).$$

Draw the straight line graph of this relation, plotting F on the horizontal axis for

$$5 \leq F \leq 75.$$

(i) Use your graph to estimate the fahrenheit temperature equivalent to a celcius temperature of 15 degrees.

(ii) Indicate on the F axis the interval $25 \leq F \leq 55$ and from the graph estimate the corresponding range for C.

(c) Find, using calculus, the co-ordinates of the local maximum of the curve

$$y = 12x - x^3.$$

7. **(a)** Differentiate

      (i) x^3

      (ii) $3x^4 - 2x + 1$

    with respect to x.

(b) Differentiate

      (i) $(x^3 + 3)(5x^2 - 1)$

      (ii) $(1 - \dfrac{1}{x})^{10}$

    with respect to x.

(c) A body moves along a straight line and its distance s metres from a fixed point on the line after a time t seconds is given by

$$s = 3t^3 - 4t + 6.$$

Find

    (i)   its speed after t seconds

    (ii)   its speed after 1 second

    (iii)   the time at which the body is at rest

    (iv)   its acceleration at $t = 3$ seconds.

8. **(a)** The function f is defined by $f : R \rightarrow R: x \rightarrow 7 - 3x$.

Find $f(-2)$, and find a number k such that

$$k f(-2) = f(24).$$

(b) Let $g(x) = x(x - 2)$ for $x \in R$.

Find $g(0)$, $g(4)$, $g(-2)$.

Show that $g(1 + t) = g(1 - t)$ for $t \in R$.

Find the derivative, $g^1(x)$, and show that $g^1(x) > 0$ for $x > 1$.

(c) Using the information obtained in (b), draw the graph of

$$g(x) = x(x - 2)$$

for $-2 \le x \le 4, x \in R$.

Using the same scales and the same axes, draw the graph of $g^1(x)$.

Write down the values of x at which the two graphs intersect.