LEAVING CERTIFICATE EXAMINATION 1978

SAMPLE PAPER (Second Draft)

Mathematics - Ordinary Level - Paper I (300 marks)

N.B. Draft paper for discussion and comments.

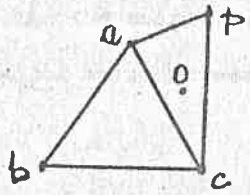
Attempt Question 1 and Four other questions

1. (i) abcd is a square field. A student takes 3 minutes to cross from a to c while walking at a speed of 4km per hour. Calculate the area of the field in square metres.

(ii) If $t = 2\pi\sqrt{\frac{\ell}{g}}$, express ℓ in terms of t and g.

(iii) If h is the orthocentre of any Δ xyz, find the orthocentre of the Δ yhz.

(iv) abc is an equilateral triangle and p is any point on the line perpendicular to bc (as in diagram). If o is the centre of the circle through the points p,a,c, prove that pao is also an equilateral triangle.



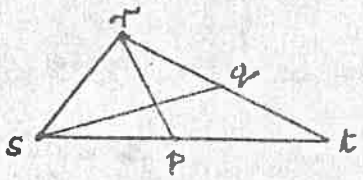
(v) Find the image of the point (-3, -2) under the translation $(0,0) \rightarrow (3,2)$.

(vi) Calculate the distance between the points $(-9, 3)$ and $(3, -2)$.

(vii) A radius of a circle is 5 units long. Express in radians the size of the angle subtended at the centre of the circle by an arc 15 units in length.

(viii) p and q are the midpoints of two sides of the Δ rst, as in diagram. Express as one vector

$$\overrightarrow{sr} + \frac{2}{3}\overrightarrow{rp} - \frac{1}{3}\overrightarrow{qs}.$$



(ix) If $3 : x = 6 : y$, find the ratio $y : x$.

(x) In a Δ abc, $|ab| = 4$cm, $|bc| = 5$cm, $|\angle abc| = 30^\circ$.

Calculate its area.

(100 marks)

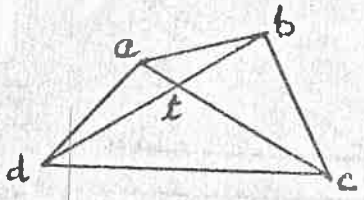
2. A cylindrical vessel of internal diameter 12cm in length is partly filled with water. A metal sphere of radius 3cm. in length is then totally immersed in the water. How high will the water rise?

(40 marks)

3. Prove that the areas of two triangles of equal height are proportional to the lengths of their bases.

In the quadrilateral abcd the diagonal [bd] is bisected at t and $|at| : |tc| = 1 : 4$

Prove that the triangles adt and abt are equal in area and find the ratio area of abcd : area of Δ adt.

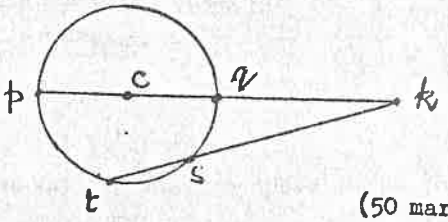


(40 marks)

4. [ab] and [cd] are two chords of a circle. If the lines ab and cd intersect outside the circle at k, prove that

$$|ak| \cdot |kb| = |ck| \cdot |kd|.$$

c is the centre of the circle, as in diagram, and q is the midpoint of [kp]. If $|ks| = 16$cm and $|st| = 2$cm, calculate the length of a radius of the circle.



(50 marks)

5. Write down the equation of the line K which contains the point p(1, 0) and which has slope 1. If K intersects the line $x = 4$ at q, find the coordinates of q.

The line T through the point q is perpendicular to K and cuts the x-axis ar r. Calculate the area of the Δ pqr.

(50 marks)

6. S is the circle $x^2 + y^2 = 25$. Verify that the point k(3, -4) is on S.

Find the equation of the circle which has k as centre and has a radius equal in length to the length of the diameter of S.

By using a central symmetry, or otherwise, find the coordinates of the point that is on both circles and verify your answer.

(50 marks)

7. (a) The function $f : x \to \sin 2x$ is defined for $x \in \mathbb{R}$.

What is meant by saying that f is periodic?

Draw the graph of f in the domain $-\pi \leq x \leq \pi$.

Use your graph to find the set of solutions of

$$1 + 2 \sin 2x = 0 \text{ for } -\pi \leq x \leq \pi.$$

(b) Two ships G and H sailed from the same port at the same time. G sailed in the direction 42° South of West (i.e. on a bearing of 228°) while H sailed in the direction 58° South of East (i.e. on a bearing of 148°). After a certain time the two ships were 5km apart and G was then due West of H. How far had the ship H sailed? Give your answer correct to the nearest kilometre.

(50 marks)

8. (a) If $\vec{a} = 3\vec{i} - 8\vec{j}$ and $\vec{b} = 4(2\vec{i} + \vec{j})$, where $\vec{i}$ and $\vec{j}$ are perpendicular unit vectors, evaluate $|\vec{ab}|$.

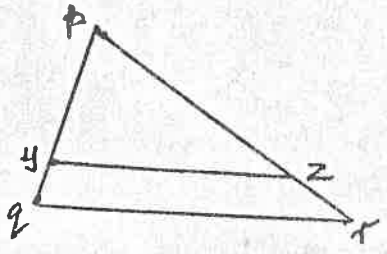
(b) In the Δ pqr

$|py| : |yq| = 3 : 1 = |pz| : |zr|$.

(i) Express $\vec{yz}$ in terms of $\vec{qy}$ and $\vec{qz}$.

(ii) Express $\vec{qz}$ in terms of $\vec{qp}$ and $\vec{pr}$.

Using (i) and (ii) deduce that yz ∥ qr.



(50 marks)