SPECIMEN PAPER II – Set B
Issued in 1969–1970

1. Find, to the nearest penny, the compound interest on £240 for 2 years at $4\frac{1}{2}\%$ per annum. Use the compound interest formula to find approximately the sum of money which would amount to £1,000 in 12 years at 6% per annum.

2. (a) Two brothers David and William begin work in 1970 on commencing salaries of £800 and £1,000 per year respectively. David's salary rises by £70 per year and William's by £50 per year. When will both brothers have the same yearly salary? How much more than David will William have earned in the meantime?

   (b) In the geometric series
   $$\frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \ldots\ldots$$
   find an expression for the 20th term and show that the sum of 20 terms is $\frac{1}{2} - \frac{1}{2^{21}}$. What is the limit of the sum of n terms as n increases indefinitely?

3. (a) Define a complex number. Explain why the set of real numbers is a subset of the set of complex numbers. What is the identity element for the addition of complex numbers? What is the additive inverse of $3 - 4i$? Express the multiplicative inverse of $3 - 4i$ in the form $a + bi$.

   (b) $z_1 = 2 - i$ and $z_2 = 2 + 4i$. Express $z_1 . z_2$ in the form $a + bi$, $a, b \epsilon$ R. What is $|z_1|$? Show that $|z_1 . z_2| = |z_1| . |z_2|$.

4A.(a) A = {1, 2}, B = {1}, C = {2}, D = ∅.
Draw up a Cayley table for the binary operation of intersection on {A, B, C, D}.
Is there an identity element ?   If so, what is it ?   Which element has an
inverse ?
Why is the set {A, B, C, D} not a group under intersection ?

(b) For an equilateral triangle show that the set of rotations about the centroid
mapping the triangle onto itself forms a commutative group under composition.

<center>OR</center>

4B.(a) The table shows the number of graded potatoes per stalk in 100 stalks.

no. of potatoes per stalk	2	3	4	5	6	7	8	9
no. of stalks	4	9	14	25	26	12	5	5

Draw a histogram to represent the distribution.   What is the median number of
graded potatoes per stalk ?   Explain your answer.

(b) Show that the mean of the distribution given by the following table is 3, and find
the standard deviation:—

x	1	2	3	4
frequency	1	4	9	6

5. (a) R is the relation {(a,a), (a,b), (a,c), (b,b), (b,c), (c,c)} on the set {a, b, c}.
Giving reasons, say whether the relation is
(i) reflexive,   (ii) symmetric,   (iii) transitive,   (iv) an equivalence relation,
(v) an order relation ?

(b) f = {$(1, 2)$, $(2, 3)$, $(3, 4)$, $(4, 1)$}; g = {$(1, 4)$, $(2, 0)$, $(3, 1)$, $(4, 2)$}.
List the elements of (i) $g \circ f$, (ii) $f \circ g$.
For what value of x is $g \circ f(x)$ = 1.   For what y is $f \circ g(y) = y$?

6. (a) f is the map $x \to f(x) = 12 - x - x^2$.
For what values of x is (1) $f(x)$ = 0,   (ii) $f(x)$ = —8 ?
What is the maximum value of $12 - x - x^2$ and what value of x gives this maximum ?

(b) If $x \in$ {—3, —2, —1, 0, 1, 2, 3}, write down the sets of values of x for which:—
(i) $3 - 2x \geqslant 5$,   (ii) $\frac{1}{x} > -\frac{1}{2}$,   (iii) $x > 2x - 3 \geqslant x - 4$.

7. f is the map $x \to f(x) = x^3 - 2x^2 - 5x + 6$.   What is $f(3)$, $f(0)$, $f(-1)$.
Find the solution set of {$x | x^3 - 2x^2 - 5x + 6 = 0$}.
Graph the function {$(x,y) | x^3 - 2x^2 - 5x + 6 = y$}, $x \in$ R, and trace the changes in value
and sign of y for the domain $-2 \leqslant x \leqslant 3$.

8. (a) $\log_{10} 2 = x$ $\log_{10} 3 = y$.   Express in terms of x and y
(i) $\log_{10} 6$,   (ii) $\log_{10} 24$,   (iii) $\log_2 3$,   (iv) $\log_{10} \sqrt{8}$

(b) Show that
$$2^{\frac{1}{2}} \cdot 3^{-\frac{1}{2}} + 3^{\frac{1}{2}} \cdot 2^{-\frac{1}{2}} = \frac{5\sqrt{6}}{6}.$$

9. (a) Find the differential coefficient of $x^3 + 1$ from first principles.
What is the value of the differential coefficient when x = 1 ?   Interpret your
result geometrically.

(b) $f : $ R $\to$ R $: x \to x^3 + 1$.   Show that $f(x)$ increases as x increases.

(c) Evaluate $\int_1^2 (3x - 1)^2 dx$.

10. (a) Differentiate $\dfrac{2x - 3}{x^2 - 2x + 3}$

(b) The distance s ft. travelled in time t secs. by a stone thrown vertically
upwards is given by
$$s = 16(2t - t^2)$$

Show that the stone stops after 1 second having travelled 16 feet.   Show that the
rate of change of the velocity is constant at all times.