SPECIMEN PAPER I - Set B
Issued in 1969-1970

1. A vessel in the shape of a cone is 5 cm. in diameter and 6 cm. deep. How much water will it hold ?

   The vessel is filled with water and the contents then poured into a cylindrical vessel of base diameter 4 cm. What is the depth of water in the cylindrical vessel ?

2. $x - 3y + 7 = 0$ is a straight line. Find:
   (i) the equation of a parallel line which contains (1,6);
   (ii) the equation of the line perpendicular to $x - 3y + 7 = 0$ which contains (1,6);
   (iii) the intersection of this perpendicular and $x - 3y + 7 = 0$, and hence the distance from (1,6) to $x - 3y + 7 = 0$.

3A. (i) Find the equation of the circle whose centre is at (0,0) and which contains (2,1).
   (ii) Find the points of intersection of the circle $x^2 + y^2 = 10$ and the line $x + y = 4$.
   (iii) Show that the point (4,3) is a point of the circle $x^2 + y^2 = 25$ and find the equation of the tangent to the circle at (4,3).

OR

3B. $[ab]$ is a line segment. Show how to find a point $c \in [ab]$ such that $|ab| \cdot |bc| = |ac|^2$. Give proof.
   If $|ab| = 2$ cm., evaluate $|ac|$ and $|cb|$.

4. abc is a triangle. The internal and external bisectors of angle A meet $[bc]$ and $[bc]$ produced at d and e respectively. Prove that

$$\frac{|bd|}{|dc|} = \frac{|ab|}{|ac|} = \frac{|be|}{|ec|}.$$

5. (a) Prove that the composite of two translations is a translation.
   (b) Show by a diagram that the composition of translations is associative.

6. (a) S_a is a central symmetry in Π. What point in Π is its own image under S_a ?
   (b) Draw a diagram to show that the set of all axial symmetries is not closed under composition.
   (c) S_A is an axial symmetry in a line A. Explain why S_A is its own inverse.
   (d) Show that a central symmetry is a bijection but that a parallel projection is not a bijection.

7. (a) p, q, r and s are distinct points $\in \Pi$, no three being collinear. Draw separate diagrams to illustrate

   (i) $\vec{pq} + \vec{qr}$, (ii) $\vec{pq} - \vec{qr}$, (iii) $\vec{pq} + \vec{qr} + \vec{rs}$.

   (b) The coordinates of two points a and b are (−2, 1) and (1, −2) respectively. Express $\vec{ab}$ in terms of $\vec{i}$ and $\vec{j}$.
   (c) If $k(3\vec{i} - 2\vec{j}) + l(-2\vec{i} + \vec{j}) = 5\vec{i} - 4\vec{j}$, find the scalars k and l.

8. Using the same axes and the same scales
   (i) indicate the set A of couples (x,y) which satisfy $\{(x,y) | x + y \leq 3\}$,
   (ii) indicate the set B of couples (x,y) which satisfy $\{(x,y) | x - y \geq -1\}$,
   (iii) indicate the set C of couples (x,y) which satisfy $\{(x,y) | y \geq \frac{1}{2}\}$,
   (iv) indicate the set D of couples (x,y) which satisfy $\{(x,y) | (x,y) \in A \cap B \cap C\}$,
   (v) find a couple $(x,y) \in D$, for which $2x + y$ has its maximum value,
   (vi) find a couple $(x,y) \in D$, for which $2x + y$ has its minimum value.

9. (a) Prove that $\cos(A + B) = \cos A \cos B - \sin A \sin B$ for all angles A and B. Given $\cos A = 0{\cdot}6$, find $\cos 2A$.
   (b) Two sides of a triangular field are 30 metres and 40 metres respectively, and the angle between them is 121° 37′. Find the perimeter and the area of the field.

10. Using the same origin and axes draw the graphs of the functions f and g where $f(x) = \cos x$ and $g(x) = 1 + \cos x$, $0 \leq x \leq 4\pi$.
   State the period and the range of $\cos x$ and $1 + \cos x$, $x \in R$.
   From your graph find the values of x between 0 and 4π for which $\cos x = 0{\cdot}5$.