L P Maths

SPECIMEN PAPER II - Set A
Issued in 1969-1970

1. Find, to the nearest penny, the amount at compound interest of £830 for 3 years at 6% per annum.

If you use the formula $A = P(1 + \frac{r}{100})^n$ and logarithms to find this same amount, what is the error ?

2. (a) The n^{th} term of a sequence is $3n-1$. Write down the 1^{st}, 2^{nd} and $(n+3)^{rd}$ terms. Find the sum of the first 20 terms of the sequence.

(b) What is meant by the sum to infinity of a geometrical series ? Under what condition does such a sum exist ?
Write the recurring decimal $0{\cdot}343434\ldots\ldots$ as an infinite G.P. and hence express the number in the form $\frac{a}{b}$ where a and $b \in n$.

3. (a) Represent the complex numbers $Z = 2 - 3i$ and $w = 1 + 2i$ by an Argand diagram. What is (i) $|z|$, (ii) $|z + w|$?

Express (i) zw (ii) $\frac{1}{z}$ in the form $a + bi$, $a, b \in R$.

(b) $(3 - 5i) - (x + yi) = (6 + i) + (y - xi)$, $x, y \in R$. Find x and y.

4A. (a) Is the set Z of integers closed under addition ? Illustrate the associativity of addition in Z by an example.
What is the identity element for addition in Z ? What is the additive inverse of (i) 5, (ii) -3, (iii) 0? Has every element in Z an additive inverse ?
Does the set Z form a group under addition ? Give reasons.

(b) Draw up a Cayley table for multiplication, modulo 3, with 0 excluded. Is the set $\{1, 2\}$ a group under multiplication in arithmetic, mod. 3 ? Give reasons.

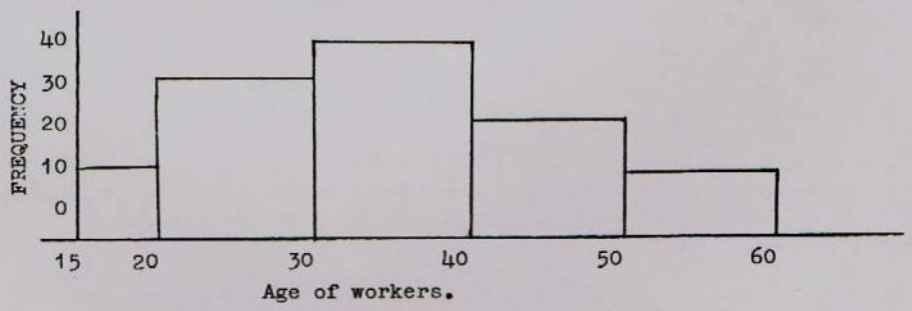
or

4B (a) What is the mean of the following array of numbers

5, 6, 11, 11, 13, 14 ?

What is the standard deviation ?

(b) The distribution of the ages of workers in a factory is shown in the histogram:-



Complete the table

Age of workers	15–20	20–30	30–40	40–50	50–60
No. of workers		30			

Assuming that the age of each worker in a class interval is at the mid-point of that interval e.g. the mid-point of the 20–30 age interval is 25, calculate the mean age of the workers.

5. (a) Define (i) a relation (ii) a function (iii) a bijection.

$R = \{(a,b), (a,c), (b,b), (b,d), (b,a), (c,e)\}$.

What is the domain and what is the range of R ? What is R^{-1}, the inverse relation ? Select from R, a subset containing <u>three</u> elements (i) which is a function but whose inverse is not a function (ii) which is bijective.

(b) $g : R \to R : x \to x^2$. $h : R \to R : x \to x + 5$. R is the set of reals.

What number is (i) $g(3)$, (ii) $h(-2)$, (iii) $g(h(4))$?

For what x is $g(h(x)) = h(g(x))$?

6. (a) Find the solution set of $\{x \mid 6x^2 - 5x - 6 = 0,\ x \in R\}$.

   (b) Find the maximum value of $4 + 3x - x^2$.

   (c) By sketching the graph of $f : f(x) = (2 - x)(2 + x)$, or otherwise, find the domain of x for which $(2 - x)(2 + x) \geqslant 0$.

7. (a) Find the solution set of $\{z \mid z^3 - 5z^2 + 4z + 10 = 0,\ z \in C\}$. Note: C is the set of complex numbers.

   (b) Draw the graph of the function $\{(x,y) \mid y = x^3 - 3x - 1\}$ for $-2 \leqslant x \leqslant 2\frac{1}{2}$. Use your graph to solve the equation $x^3 - 3x - 1 = 0$.

8. (a) Use the Binomial theorem to expand $(1 - 2a)^5$.

   Find the value of $(0{\cdot}98)^5$ correct to 4 significant figures.

   (b) Find the least value of n for which

$$\frac{2}{n+1} < 0{\cdot}01,\ n \in N.$$

   What is $\lim_{n \to \infty} \frac{2}{n+1}$?

9. Differentiate $x^2 - 3x + 2$ with respect to x from first principles.

   At what point on the curve $y = x^2 - 3x + 2$ is the slope of the tangent = 1 ?

   What is the equation of the tangent at that point ?.

10. (a) Differentiate (i) $(2x^2 - 3x - 1)(x^3 - x + 2)$ (ii) $\frac{3x}{2x+1}$

    (b) Evaluate

$$\text{(i)} \int_1^2 x^2\,dx \quad \text{(ii)} \int_0^2 (x^3 - 3x + 1)\,dx.$$