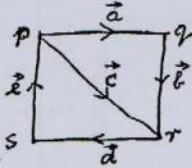## SPECIMEN PAPER I – Set A
Issued in 1969–1970

1. What is the volume of a sphere of diameter 5 cm ?
   A cylindrical tank diameter of base 16 cm contains water. Four spheres, each of diameter 5 cm, are placed in the tank and wholly submerged. What is the increase in the height of the water level ? Give your answer correct to two significant figures.

2. a and b are two points the coordinates of which are $(-3, 1)$ and $(3, 9)$ respectively. Find (i) the length of $[ab]$, (ii) the equation of ab, (iii) the equation of the line perpendicular to ab which contains the mid-point of $[ab]$, (iv) the points where ab cuts the axes.

3A. (i) What is the radius of the circle $4x^2 + 4y^2 = 25$?
   (ii) State whether each of the following points is within, belongs to, or is outside this circle:- $(0, 0)$, $(0, \frac{1}{2})$, $(1, 1)$.
   (iii) Find the points of intersection of this circle and the line $2x + 2y = 5$.

<div align="center">OR</div>

3B. Show, without proof, how to construct an isosceles triangle abc with $\angle abc = \angle acb = 2\angle bac$. How many degrees in $\angle abc$?
   C is a circle centre O. Points a, b, c, d, e are elements of C such that $\angle aob = \angle boc = \angle cod = \angle doe = \angle eoa$. Prove that figure $abcde$ is a regular pentagon.

4. Prove that the bisectors of the angles of a triangle are concurrent.
   $abcd$ is a quadrilateral. The bisectors of the angles at a, b and c meet at a point x. Prove that the bisector of the angle at d also passes through x.

5. (i) a, b are two points. x is another point which is not an element of ab.
   $S_b \circ S_a(x) = y$. Show that ab is parallel to xy.
   (ii) Prove that the composite of two central symmetries is a translation.
   (iii) Show by a diagram that the composition of central symmetries is not commutative.

6. (i) A and B are two lines and $A \cap B \neq \emptyset$. Illustrate by a diagram the rotation $S_A \circ S_B$.
   When is $S_A \circ S_B = S_B \circ S_A$?

   (ii) C and D are lines; $C \parallel D$ and $C \neq D$.

   Draw a diagram to illustrate the map $S_C \circ S_D$. Prove that this map is a translation. Is $S_C \circ S_D = S_D \circ S_C$?

7. (a) $pqrs$ is a square.
   (i) Express each vector below as simply as possible (see diagram)
   $\vec{a} + \vec{b}$; $\vec{a} + \vec{b} + \vec{d}$; $\vec{a} + \vec{b} + \vec{d} + \vec{e}$.
   (ii) If the norm of $\vec{a} = 3$, what is the norm of $\vec{c}$?
   (iii) If x is the mid-point of $[p\ r]$, express $\vec{qx}$ in terms of $\vec{a}$ and $\vec{c}$
   (b) If $\vec{x} = 2\vec{i} + \vec{j}$ and $\vec{y} = -2\vec{i} - \vec{j}$, what is $2\vec{x} - 3\vec{y}$ in terms of $\vec{i}$ and $\vec{j}$?
   Evaluate $|\ 2\vec{x} - \vec{y}\ |$.

8. A factory makes two models of a product, a standard model and a deluxe model. The standard model is made in 8 minutes on machine A followed by 8 minutes on machine B. The deluxe model is made in 6 minutes on machine A followed by 12 minutes on machine B. Machine A can operate for only 72 minutes over a given period and machine B for only 96 minutes in the same period.
   If x is the number of standard and y the number of deluxe models made over this given period, justify the constraint $8x + 6y \leqslant 72$. What are the other constraints ? The profit on a standard model is £40 and on a deluxe model is £50. How many models of each kind should the factory turn out in the given period to secure a maximum profit ? What is the maximum profit ?

9. (i) What is the measure in radians of the angle subtended at the centre of a circle, radius 3 cm by an arc of length 5 cm.
   (ii) If $\sec x = \frac{5}{4}$, find $\cos x$ and $\sin 2x$ without using tables.
   (iii) The sides $[ab]$ and $[ac]$ of a triangle abc are equal in length. $|\ bc\ | = 6$ metres and $\angle abc = 30°$. Find the area of triangle abc in sq metres correct to one decimal place.

10. State the period and the range of f; $f(x) = \sin 2x$, $x \in R$.
   Graph f in the domain $0 \leqslant x \leqslant 2\pi$.
   If $g(x) = 3 \sin x$, what is the range of g for $x \in R$.
   Write down a periodic function whose period is 2π and whose range is $[-\frac{1}{2}, \frac{1}{2}]$.