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LEAVING CERTIFICATE EXAMINATION, 1994

MATHEMATICS — HIGHER LEVEL

SAMPLE PAPER I (300 marks) - 2½ hours

Attempt **SIX QUESTIONS** (50 marks each)

Marks may be lost if all your work is not clearly shown or if you do not indicate where a calculator has been used.

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1. **(a)** Show that

$$\frac{x+3}{x-1} + \frac{4}{1-x}$$

simplifies to a constant for $x \neq 1$.

(b) Solve the simultaneous equations

$$2x + 3y - z = -7$$
$$5x - 2y - 4z = 3$$
$$3x + y + 2z = -7$$

(c) Prove that if $f(x)$ is a cubic polynomial and k is a number such that $f(k) = 0$, then $x - k$ is a factor of $f(x)$.

Let $g(x)$ be a cubic polynomial such that

$$g(-1) = 0,\ g(1) = 0 \text{ and } g(2) = 5g(0).$$

Find the third root of the equation $g(x) = 0$.

2. **(a)** Find the two solutions for the linear and quadratic equations

$$y = x - 1$$
$$x^2 + y^2 = 13$$

(b) If $a \neq 0$ and one of the roots of the equation $ax^2 + bx + c = 0$ is three times the other, show that

$$3b^2 = 16ac.$$

(c) If for all integers n,

$$U_n = 600(2)^n - 7(5)^n$$

verify that

$$U_{n+2} - 7U_{n+1} + 10U_n = 0.$$

Find the least integer $n > 0$ for which $U_n < 0$.

3. (a) Let $z = -1 + i\sqrt{3}$, where $i^2 = -1$. Express z^2 in the form $x + iy$, $x, y \in \mathbb{R}$ and find the real value for k such that

$$z^2 + kz$$

is real.

(b) If the matrix $M = \begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$, find M^2.

Given that $M^{-1} \begin{bmatrix} p & q \\ r & s \end{bmatrix} = M \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}$, find the values of p, q, r and s.

(c) If

$N = \begin{bmatrix} 4 & 3 \\ 3 & -4 \end{bmatrix}$ and $P = \begin{bmatrix} 3 & -1 \\ 1 & 3 \end{bmatrix}$,

calculate $P^{-1}NP$ and hence find N^{20}.

4. (a) Find the sum of the first 12 terms of the geomtetric sequence

$$2,\ 2 \times 3,\ 2 \times 3^2,\ 2 \times 3^3,\ \ldots$$

(b) In an arithmetic sequence, three consecutive terms have a sum of –9 and a product of 48.

Find the possible values for these terms.

(c) Show that the nth term of the sequence

$$5,\ 55,\ 555,\ 5555,\ \ldots$$

can be written as the sum to n terms of a geometic series and has the value $\frac{5}{9}(10^n - 1)$.
Hence find the sum of the first n terms of the sequence.

5. **(a)** Expand $(2 + \sqrt{3})^5$ by the Binomial Theorem, and write your answer in the form

$a + b\sqrt{3}, \quad a, b \in \mathbb{R}$.

(b) Show that for $n \geq 1$,

$$\sum_{r=1}^{n} \frac{1}{(r+1)(r+2)} = \frac{1}{2} - \frac{1}{n+2}.$$

Hence find the sum of the infinite series

$$\sum_{r=1}^{\infty} \frac{1}{(r+1)(r+2)}.$$

(c) Show by induction that 8 is a factor of $7^{2n+1} + 1$ for $n \in \mathbb{N}$.

6. **(a)** Find the derivative of the functions (*i*) $(2x + 1)^3$ and (*ii*) $\dfrac{x}{x^2 + 1}$.

(b) Let

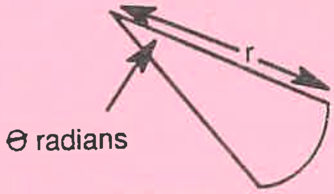
$x = 4\cos\Theta + 3\sin\Theta$ and $y = 3\cos\Theta - 4\sin\Theta$, where $-\pi < \Theta < \pi$.

Evaluate $\dfrac{dy}{dx}$ when $\Theta = \dfrac{\pi}{2}$.

(c) Let $f(x) = e^{2x} - ae^x$, $x \in \mathbb{R}$ and a constant, $a > 0$.

Show that $f(x)$ has a local minimum at a point $(b, f(b))$, specifying the value of b in terms of a.

7. **(a)** The sector of the circle shown, of radius length r, has total perimeter length 20. Express this information in an equation involving r and Θ.



Express Θ in terms of r and hence obtain an expression for the area of the sector in terms of r.

(b) **(i)** Diferentiate from first principles $\sqrt{x}$ with respect to x.

(ii) Find the derivative of $\dfrac{x}{1 + \sqrt{x}}$

(c) Let $f(x) = 2 \tan^{-1} x - \tan^{-1} \dfrac{2x}{(1 - x^2)}$, for $x \neq -1, 1, \quad x \in R$.

Find $f^1(x)$, the derivative of $f(x)$.

Show that $2 \tan^{-1} x = \tan^{-1} \dfrac{2x}{(1 - x^2)}$,

for $-1 < x < 1$.

8. **(a)** Find $\displaystyle\int 2x^3 \, dx$ and $\displaystyle\int \frac{x^3 - 2}{x^2} \, dx$

(b) Evaluate **any two** of the following

(i) $$\int_0^1 x\sqrt{1 - x^2}\, dx$$

(ii) $$\int_{\pi/3}^{2\pi/3} \sin 4x \cos 2x \, dx$$

(iii) $$\int_{2/3}^{2\sqrt{3}} \frac{dx}{4 + 9x^2}$$

(c) Using De Moivre's Theorem, prove that

$\text{Cos } 3\Theta = 4\cos^3 \Theta - 3\cos \Theta$.

Hence find

$$\int \cos^3 \Theta \, d\Theta.$$