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LEAVING CERTIFICATE EXAMINATION, 1978

MATHEMATICS - HIGHER LEVEL - PAPER II (300 marks)

SAMPLE PAPER

Attempt QUESTION 1 (100 marks) and FOUR other questions (50 marks each).

1. (i) If $z_1 = 6 + 4i$ and $z_2 = 2 + 3i$, where $i = \sqrt{-1}$, calculate $z_1 . z_2$ and $\bar{z}_1 . \bar{z}_2$.

(ii) Find from first principles the differential coefficient of $\cos x$ with respect to x.

(iii) If $x = 2t - t^2$ and $y = t^3$, where $t \in R$, find the value of $\frac{dy}{dx}$ when $t = 2$.

(iv) Evaluate $\sum_{r=1}^{n} (1 + r + 2^r)$

(v) The line segment $y = x - 1$, $1 \leqslant x \leqslant 4$, is rotated about the line $x = 0$. Find in terms of π the volume generated.

(vi) Prove by induction, or otherwise, that $1 + x + x^2 + .. x^n = \frac{1 - x^{n+1}}{1 - x}$

(vii) Assuming that $\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges, show that $\sum_{n=1}^{\infty} \frac{n x^n}{n^3 + 1}$ converges for $0 \leqslant x \leqslant 1$.

(viii) T_n is the nth term and S_n is the sum of the first n terms of a series. If $S_n = nT_n - 7$ for $n > 1$ and $T_1 = 5$, show that $T_n = T_{n-1}$ and find S_{13} .

(ix) $x = y_1 (1{\cdot}15) = y_2 (1{\cdot}15)^2$ and $y_1 + y_2 = 500$. Calculate x to the nearest integer.

(x) If $\vec{a} = 3\vec{i} - 2\vec{j}$, $\vec{b} = \vec{i} + 4\vec{j}$, $\vec{c} = 2\vec{i} + \vec{j}$, verify that $\vec{a}.(\vec{b} + \vec{c}) = \vec{a}.\vec{b} + \vec{a}.\vec{c}$.

OR

(x) Use the tables on page 36 to find the area under the normal curve which corresponds to $-1{\cdot}6 \leqslant z \leqslant 1{\cdot}6$.

2. (a) If $x, y \in R$ and $i = \sqrt{-1}$, express $\sqrt{3 + \frac{4}{i}}$ in the form $x + iy$.

(b) Show that $z_1 \bar{z}_2 - \bar{z}_1 z_2 = 2i \, Im(z_1 \bar{z}_2)$, where $Im(z)$ means the imaginary part of z.

(c) Illustrate on an Argand diagram the set of solutions of $| z - 1 - i | = 2$.

3. Let $A = \{ x \mid |x| \geqslant 2 \}$ and let f be the function $A \to R : x \to x\sqrt{x^2 - 4}$.
Show that f has no local maximum or minimum but that it has two points of inflexion.
Show also that the tangent to the graph of f at the point (2, 0) is parallel to the y - axis and prove that $f(x)$ increases with x for all $x \in A$.
Draw the graph of $y^2 = x^2(x^2 - 4)$.

4. (a) Differentiate with respect to x:

(i) $\sin 2x \cos^2 x$

(ii) $e^{\frac{1}{1 - x}}$, $x \neq 1$

(iii) $\log(\tan 3x)$, $0 < x < \frac{\pi}{6}$

(iv) 2^x .

(b) If $y = (a + bx) e^{-2x}$, where a, b are independent of x, prove that

5. Evaluate:-

(i) $\int_2^3 \log k \, dx$ (ii) $\int_0^{\frac{\pi}{2}} \sin 3x \cos x \, dx$

(iii) $\int_0^2 \frac{dt}{5 + t(t+4)}$ (iv) $\int_1^2 \frac{x}{x+1} \, dx$

6. (a) Let $T_n = \frac{(2n-1)(3n+1)}{1+n+n^2}$ and let $\lim_{n\to\infty} T_n = T$.

Find the least value of n for which $T - T_n < 0{\cdot}5$.

(b) Let $S_n = u_1 + u_2 + \ldots\ldots + u_r + \ldots\ldots u_n$ where each $u_r > 0$.

Write u_n as a difference of two sums and show that $\lim_{n\to\infty} u_n = k > 0$ implies that the infinite series $u_1 + u_2 + \ldots\ldots + u_r + \ldots\ldots$ is divergent.

Test for convergence:-

$$\frac{1+3(1^2)}{1+1^2} + \frac{1+3(2^2)}{1+2^2} + \ldots\ldots + \frac{1+3(r^2)}{1+r^2} + \ldots\ldots$$

7. Determine real numbers a, b such that $\frac{x+1}{x^2(x+2)^2} = \frac{a}{x^2} + \frac{b}{(x+2)^2}$ for all $x \in R \setminus \{0, -2\}$.

Hence, or otherwise, find a formula for the sum of the first n terms of the series

$$\sum_{r=1}^{\infty} \frac{r+1}{r^2(r+2)^2}$$ and deduce that the sum of the series is $\frac{5}{16}$.

8. The random variable x has a binomial distribution such that $\bar{x} = 12$ and $\sigma = 5$. Estimate the probability that $x \geqslant 20$.

Let x be a random variable denoting the number of heads found when an unbiassed coin is tossed 576 times. Find $\bar{x}$ and σ and using $z = \frac{x - \bar{x}}{\sigma}$ estimate the limits between which the number of heads lies so as to have a probability of 95% of being correct.

OR

8. If t is a real number and r is a point in a line ab, prove, with respect to any origin not in ab, that

$$\vec{r} = t\vec{b} + (1-t)\vec{a}.$$

If p is any point in the interior of a $\Delta\, abc$, prove that

$$\vec{p} = x\vec{a} + y\vec{b} + z\vec{c}$$

where x, y, z are positive numbers less than 1 and $x + y + z = 1$.