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LEAVING CERTIFICATE EXAMINATION, 1978

MATHEMATICS - HIGHER LEVEL - PAPER I (300 marks)

SAMPLE PAPER

Attempt Question 1 (100 marks) and FOUR other questions (50 marks each)

1. (i) Find the value of t for which the simultaneous equations $2x + 3y = 6; \quad 5x + ty = 15$ have more than one solution.

(ii) How many 4 figure natural numbers can be made from 2, 2, 4, 5, 6 ?

(iii) Without evaluating the binomial coefficients, prove that

$$\binom{20}{0}+\binom{20}{2}+\binom{20}{4}+\cdots+\binom{20}{20}=\binom{20}{1}+\binom{20}{3}+\binom{20}{5}+\cdots+\binom{20}{19}$$

(iv) Find the equations of the pair of lines $6x^2 + xy - y^2 - 14x + 3y + 4 = 0$.

(v) If the x-axis is a tangent to the circle $x^2 + y^2 + 2gx + 2fy + c = 0$, express c in terms of g.

(vi) Find $\begin{pmatrix} 1{\cdot}6 & -3{\cdot}5 \\ -2{\cdot}8 & 6{\cdot}0 \end{pmatrix}^{-1}$ and solve $\begin{pmatrix} 1{\cdot}6 & -3{\cdot}5 \\ -2{\cdot}8 & 6{\cdot}0 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$

(vii) If $A = \begin{pmatrix} 4 & 3 \\ -1 & 1 \end{pmatrix}$ and $B = A^2$, find B. Is $AB = BA$?

(viii) If $\tan a = \frac{4}{3}$, $0 < a < \frac{\pi}{2}$, find $\tan\frac{a}{2}$ without using the Tables.

(ix) Find the $\lim\limits_{x \to 0} \frac{1}{x} \sin 3x$.

(x) Let e be the identity of a multiplicative group G.
Suppose $g \in G$ such that $g^5 = g^{17} = e$. Prove that $g = e$.

OR

(x) Find the coordinates of the vertex and of the focus of the parabola $x^2 + 2x + 4y - 3 = 0$.

2. (a) Given the simultaneous equations

$$\begin{aligned} 2x - y + 2z &= 3 \\ x + 3y - z &= 2 \\ 3x + 2y + z &= t \end{aligned}$$

find a value of t for which (i) there is no solution (ii) there is an infinity of solutions. Is there a value of t for which there is only one solution ?

(b) If $(1 - i)$ is a root of the equation
$2x^3 - 5x^2 + kx - 2 = 0$, $k \in R$,
find the value of k and the other two roots.

3. Prove that $(1 + x)^n = \sum\limits_{r=0}^{n} \binom{n}{r} x^r$ for $n \in N_0$.

Write out the first 3 terms of the binomial expansion $(1 + 2x)^{\frac{2}{3}}$. If x is so small that its square and higher powers may be neglected, find an approximation of the form $a + bx$ for

$$\frac{\sqrt[3]{(1 + 2x)^2}}{4 - x}.$$

4. L_1 and L_2 are two lines which intersect on the y-axis and which make angles measuring 60° and 30°, respectively, with the positive sense of the x-axis. If the area of the region enclosed by L_1 and L_2 and the x-axis is $\sqrt{3}$ units, find the equations of L_1 and L_2.
Find also the area of the parallelogram enclosed by the four lines.

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5. P and Q are two circles which touch externally. The centre of P is (12, 5) and the equation of Q is $x^2 + y^2 = 16$. Find the equation of (i) P, (ii) the common tangent at the point of contact, (iii) the length of the segment of the x-axis cut off by P.

6. (a) If $\vec{i}$ and $\vec{j}$ are orthonormal vectors and f is a linear transformation such that $f(\vec{i}) = 2\vec{i} + \vec{j}$ and $f(\vec{j}) = \vec{i} + 2\vec{j}$, find the image of the $\Delta\, o\, a\, b$ under f where o is the origin, $\vec{a} = \vec{i} + 4\vec{j}$ and $\vec{b} = 4\vec{i} - \vec{j}$.

(b) Write down the matrix of a rotation, K, of angle θ where $0 < \theta < \frac{\pi}{2}$, about the origin.

Tan θ is the gradient of a line L which contains the origin and S_L is the axial symmetry in L. Prove that $\begin{pmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{pmatrix}$ is the matrix for S_L.

What is the image of the line $x + y \tan \frac{\theta}{2} = 0$ under $S_L \circ K$?

7. (a) Prove that $\cos(A - B) = \cos A \cos B + \sin A \sin B$.

(b) Prove $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$.

(c) Write down the period of the function f: $R \to R : x \to 3 \sin \frac{2x}{3}$. If g is a trigonometrical function such that the period of $3 \sin \frac{2x}{3} + g(x)$ is 6π, find one such function g.

8. (a) Let $A = \{0, 1, 2, 3\}$ and $B = \{1, 3, 5, 7\}$.
Prove that A is a group under addition (mod 4) and that B is a group under multiplication (mod 8).
Prove also that A and B are not isomorphic groups.

(b) Let P be the set of all subsets of a set S. Suppose that P is a group under union. Prove that the empty set is the identity element of P and hence show that S is the empty set.

OR

8. (a) Find the equation of the tangent to $y^2 = 8x$ which is parallel to the line $x = 3y$.

(b) Find (i) the slope (ii) the equation of the tangent to the parabola $y^2 = 4ax$ at the point $p\ (at^2, 2at)$. If s is the focus and pk is a line parallel to the axis of the parabola, prove that ps and pk make angles of equal measure with the tangent at p.