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SPECIMEN PAPER

LEAVING CERTIFICATE EXAMINATION

MATHEMATICS — HIGHER COURSE — PAPER I — SET A

1. The coordinates of the vertices of a triangle are $(-4, 3)$, $(0, -5)$ and $(3, 4)$. Find the coordinates of h the orthocentre.
Show that o, the origin, is the circumcentre of the triangle. If g is the centroid (the point of intersection of the medians), find the ratio in which g divides $[oh]$.

2. A circle is represented by the equation $(x + 1)(x + 3) + (y - 4)(y + 2) = 0$, find the radius and the coordinates of the centre, and show that the circle touches the straight line $3x - y + 17 = 0$.
The circle is divided into two segments by the straight line $y = 2x$; show that the angle in one segment is an angle of $45°$.

3. (a) Define a parabola. The focus of a parabola is the point $(2, 3)$ and $y + 3 = 0$ is the equation of the directrix. Find the equation of the parabola.
(b) Find the focus and directrix of the parabola $2x^2 + 6y - 10x + 17 = 0$.
(c) Find the values of m and c for which the line $y = mx + c$ is a tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

4. (a) Explain what is meant by saying that the scalar product of two vectors is (i) bilinear, (ii) positive definite.
(b) Let p, q, r, s be the lengths of the sides of a quadrilateral where the side of length p is opposite the side of length r.
Prove that $p^2 + r^2 = q^2 + s^2$ if and only if the diagonals are perpendicular.

5. (i) $\vec{a}$ and $-\vec{a}$ are the end points of a diameter of a circle with centre the origin and radius r. If $\vec{p}$ is any point of the plane, prove that
$(\vec{p} - \vec{a}) \cdot (\vec{p} + \vec{a}) = |\vec{p}|^2 - r^2$.
(ii) $\vec{x}$, $\vec{y}$, $\vec{u}$, $\vec{v}$ are points of a circle with centre the origin. The lines $\vec{x}\,\vec{y}$ and $\vec{u}\,\vec{v}$ intersect in $\vec{p}$.
Prove that $|\vec{p} - \vec{x}|\,|\vec{p} - \vec{y}| = |\vec{p} - \vec{u}|\,|\vec{p} - \vec{v}|$.

6. (a) Define a linear transformation of the pointed plane Π_0.
(b) Prove that under a linear transformation the image of a line is either a line or a point.
(c) If f is a linear transformation and $\mathcal{L}$ is the set of lines of the plane, prove that $A, B \in \mathcal{L}$ and $f(A), f(B) \in \mathcal{L}$ and $A \| B$ implies $f(A) \| f(B)$.
[Hint: if $\vec{a}_1, \vec{a}_2 \in A$ and $\vec{b}_1, \vec{b}_2 \in B$ then $\vec{a}_1 - \vec{a}_2 = k(\vec{b}_1 - \vec{b}_2)$ $k \in R$.]

7. Write down the matrix (with respect to an orthonormal basis) of the rotation r of the plane with centre the origin which maps the point $(1, 0)$ on to the point $\left(\frac{3}{5}, -\frac{4}{5}\right)$.
What is the image of $(5, 20)$ under r ? Of what point is $(4, 18)$ the image ? Find the measure (in degrees between $0°$ and $360°$, to the nearest degree) of the angle of this rotation.

8. Define a periodic function. If the function f has a period k prove that $2k$ is also a period and that $-k$ is a period.
Find the least positive period of $x \to \text{Sin}\,x$, $x \to \text{Sin}\,\frac{\pi x}{3}$, $x \to 7\,\text{Cos}\left(5 - \frac{3\pi x}{2}\right)$, $x \to \text{Sin}\,x + \text{Cos}\,x$.

9. (i) Define the conjugate of a complex number.
(ii) If z_1 and z_2 are complex numbers prove that $\overline{z_1 + z_2} = \bar{z}_1 + \bar{z}_2$ and $\overline{z_1 z_2} = \bar{z}_1 \cdot \bar{z}_2$.
(iii) If $f(z) = a_0 z^n + a_1 z^{n-1} + \ldots\ldots + a_{n-1} z + a_n$ where $a_0, a_1 \ldots\ldots a_n \in R$ and $z \in C$ prove that $\overline{f(z)} = f(\bar{z})$.
(iv) If w is a root of the equation $f(z) = 0$ prove that $\bar{w}$ is also a root.

10. Let $R_0 = R \setminus \{o\}$ and $G = R_0 \times R$. An operation $*$ is defined on G as follows:
$(a, b) * (c, d) = (ac,\ bc + d)$. Prove G, $*$ is a group. Is the group commutative ? What is the inverse of $(5, 3)$?

OR

10. (a) If p_1, p_2, p_3 are the probabilities of the events E_1, E_2, $E_1 \cap E_2$ respectively, what relation connects p_1, p_2, p_3 when (i) E_1, E_2 are independent events (ii) E_1, E_2 are not independent events ?
Two cards are drawn at random from a pack of 52 cards. Find the probability that both cards are aces if (i) the first card picked is replaced (ii) the first card picked is not replaced.
(b) Two boys A and B toss an unbiassed penny and the first to obtain a "head" wins. If A has first toss find the probability that (i) A wins on his first toss (ii) B wins on his first toss (iii) A wins on his second toss. Calculate the probability that A wins.