

MATHEMATICS - HIGHER COURSE - PAPER II

MONDAY, 16th JUNE - Morning 9.30 to 12

Six questions to be attempted.

All questions carry equal marks.

Mathematical tables may be obtained from the superintendent.

N is the set of natural numbers.

Z is the set of integers.

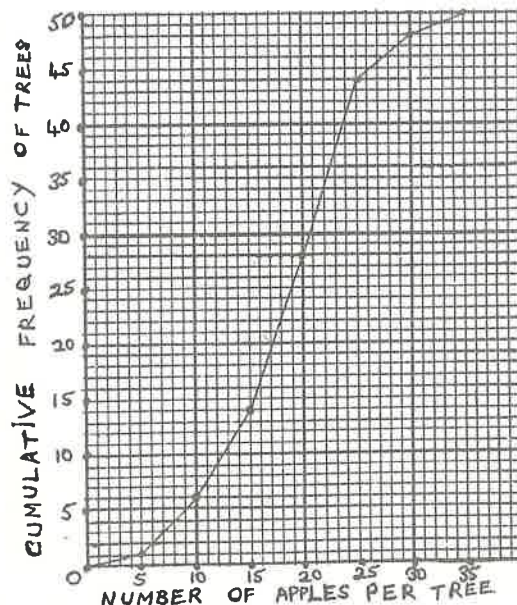
R is the set of real numbers.

1. To how much will £250 amount if invested for 2 years at $7\frac{1}{2}\%$ per annum, compound interest? Give your answer in £, correct to 2 places of decimals.
How much money should be invested for 2 years at $7\frac{1}{2}\%$ per annum, simple interest to give this same amount? Give your answer to the nearest £.
2. (a) 230 students participated in a school sports; 45 students entered for the long jump only while 19 students entered for the high jump only. If 156 students did not take part in either of the two jumps, how many students entered for both jumps?
(b) By considering the sets A, B, C, write an equation (one in each case) to illustrate that
(i) Union of sets is commutative.
(ii) Union of sets is associative.
If $B \cup A = C$, prove that $A \cup (B \cap C) = C$.
3. Let $X = \{2, 20, 200\}$. Graph the relation \leq defined on X and write down the elements of the domain and image (range) of the relation.
Is the relation (i) transitive (ii) symmetric (iii) a map? Give your reason in each case.
4. 300 notebooks were divided equally among a class of children. If each child had been given 2 notebooks less, there would be enough notebooks left over for 5 other children. How many children were in the class?
5. (a) The first and second terms of each of two sequences are 1 and 2, respectively, one of the sequences being arithmetic and the other being geometric. Find the 3rd and 50th term of each sequence.
Find also the sum of 50 terms of the arithmetic sequence.
(b) The nth term of a sequence is $n^2 - 2n + 2$. Write down the first five terms of this sequence.
6. (a) Divide the binary number 1,110,101 by the binary number 1,001 and express your answer in binary form.
(b) If $y = x\sqrt{1-x^2}$, evaluate y correct to the nearest integer where $x = 14.08$ and $\pi = 0.67$.
7. (a) Write down the factors of $14 - x^2 + 13x$ and hence find the solution set of
 $\{x \mid x^2 = 13x + 14, x \in \mathbb{Z}\}$
(b) Graph the relation
 $\{(x, y) \mid y = 4x^2 - 4x - 3, (x, y) \in \mathbb{R} \times \mathbb{R}\}$
taking $-1 \leq x \leq 2$ as the domain of the relation.
From your graph, show that the line $x = \frac{1}{2}$ is an axis of symmetry of the graph and hence, or otherwise, estimate the minimum value of $4x^2 - 4x - 3$ ($x \in \mathbb{R}$).
8. Let * be defined by $a * b = \frac{1}{2}ab$ ($a, b \in \mathbb{R}$).
Answer each of the following giving reasons for your answers.
(i) Is * commutative?
(ii) Is * associative?
(iii) Is the set N closed under *?
(iv) Is the set of positive rational numbers closed under *?
(v) Solve the equation $5 * x = 1$, $x \in \mathbb{R}$.

9. The number of apples harvested from each tree in an orchard was recorded and a cumulative frequency graph was drawn of the number of trees giving less than 5 apples per tree, less than 10 apples per tree etc. The diagram shows the graph.

From the graph find

- (i) The number of trees in the orchard.
- (ii) The number of trees giving 15 (or more) but less than 25 apples per tree.
- (iii) The median of the distribution of apples.



10. Using the same axes and the same scales draw the lines

$$x = 1, \quad y = 1, \quad x + y = 3.$$

Indicate the set of points (x, y) which simultaneously satisfy

$$0 < x < 1, \quad y \geq 1, \quad x + y \leq 3.$$

If (x, y) is an element of this set, find the point (x, y) for which $x + 2y$ is maximum.