LEAVING CERTIFICATE MATHEMATICS COURSES

RATIONALE AND CHANGES

INTRODUCTION

This document provides a summary of the rationale informing the design of the new Leaving Certificate Mathematics courses, and also lists the main alterations (inclusions and exclusions) in the Higher and Ordinary courses as compared with their predecessors.

THE HIGHER COURSE

The Higher course serves those who are aiming to specialise in areas such as mathematics itself, the physical sciences, engineering and technology; but it must also provide a good general education for able students who do not intend to proceed further with the study of mathematics. In terms of style and content, therefore, the main need appeared to make the course more attractive and accessible without radically changing the topic areas addressed.

This has been achieved in the main by two approaches, namely the following:

- reducing the length of individual topics, either by excising aspects which had dated or which seemed unproductive in terms of educational value or usefulness, or by truncating long topics;
- removing entire topics.

In general, it was decided that the course should remain broad, with various different types of topic being retained — hopefully appealing to a variety of interests, needs and learning styles. Thus, the first approach was the more favoured. In particular, topics have been truncated by trying to avoid using the most advanced techniques from one area in order to solve problems in other areas. (Naturally, this does not mean that the idea of integrating mathematical topics is opposed in principle; rather, for the most difficult aspects of the course, there is insufficient time for students to develop the required maturity to implement it in practice.) Hopefully the course still has enough depth to challenge and develop the most able students, but will not outface the less gifted.

Naturally there are also some new elements as regards content. Due attention is given to topics that appear to be both of increasing prominence in mathematics itself and — more importantly — relevant and suitable for study at school level. In particular, some rather practical work on *probability and statistics* has been put into the core, and the area of *discrete mathematics* has been given greater importance overall. (Problems as regards resources obviated much attention being paid to *numerical methods*: time constraints prevented even the small amount in the Ordinary level courses being reproduced in the Higher course. And while on the subject of omissions, it may be noted that the course does not contain the "accurate drawing" aspect present in the two Ordinary level courses; however, there is some material with visual appeal and involving spatial skills.)

One other point should be mentioned. As proof is a major concept in mathematics and one which students at this level hopefully are able to grasp and appreciate (following their introduction to it via theorems in synthetic geometry in the Junior Certificate Higher course), different kinds of proof have been featured in the syllabus. In fact, some element of proof has been introduced in almost every section.

CORE

Algebra

The algebraic topics in the syllabus have been brought together under a single heading, and emphasis has been put on the basics in the hope of helping students to build a firm foundation for their algebraic work.

Material omitted as compared with the previous course:

- Basics: Remainder Theorem (but see Factor Theorem); Horner 's method (but see Newton-Raphson under Calculus); sums and products of roots of cubic equations.
- Simultaneous equations: non-unique solutions.
- Complex numbers: properties of the modulus; loci.
- Matrices: mandatory application to linear transformations.

New Material:

- Quadratic and rational inequalities and $|x - a| = b$.

Geometry

All three elements — co-ordinate geometry, vectors, and transformation geometry — are represented. However, the treatment of vectors is minimal. It is intended that the approach might be motivated by applied mathematics (thus, a starting point might be a plane or ship blown off course by a strong wind); and the development is intended to be intuitive, leading on this occasion to algebraic manipulation rather than to proof of geometric theorems. The intention is to introduce students briefly to a major useful concept — partly because of its general educational role, and partly to give those proceeding to further study a "first pass" over the topic — without occupying too much time.

The changed approach to vectors necessitated a new approach to transformation geometry. It builds on the study of isometries in the Junior Cycle Higher course, moving on to consider ideas of invariance or non-invariance under transformations.

Material omitted as compared with the previous course

- Line: pairs of lines
- Circle: intersection of line and general circle; systems of circles.
- Vectors: "abstract" approach, based on translations; theorems.
- Transformation geometry: presentation via vectors; sundry results.

New material:

- Circle: parametric equations of circles.
- Vectors: $r^{\perp}$
- Transformation geometry: presentation via mappings given in co-ordinate form; sundry results.

Trigonometry

Somewhat more explicit emphasis is given to techniques at the expense of advanced theory in this area, to give students a firm foundation for applications.

Material omitted as compared with the previous course:

- Inverse functions: compositions; sum of inverse tangents (but see option).
- Periodicity: sums and products.
  (Periodicity is given a limited treatment — see under Functions.)

Sequences and Series

Material omitted as compared with the previous course:

- Binomial series: theorem for rational but non-natural exponent; approximations.
- Limits: recurrence relations (but see discrete mathematics).
- Convergence tests (but see option).

New material:

- Recurring decimals considered as infinite series.

Functions and Calculus

The study of functions, and in particular of the differential and integral calculus, has traditionally been a key aspect of mathematics. It remains important: but with the availability of information technology — as regards both handling manipulations in this area, and allowing the growth of other areas of mathematics such as numerical methods — the value of developing skills in calculus is somewhat reduced. This is reflected in an intention to lighten the load as regards complicated examples.

Material omitted as compared with the previous course:

- Differentiation: second derivative of parametric functions; problems involving modelling (but see option).
- Integration: functions not of specified forms; volumes of revolution other than cones and spheres.

Discrete Mathematics and Statistics

By contrast to the previous area, this is one that has been growing in importance within mathematics. Difference equations provide an analogue to differential equations for continuous functions: they have many applications in natural science. (They also lend themselves to computerised treatment, but this aspect is omitted from the course.) In the case of probability and statistics, their high profile in everyday life provides another reason for including at least a simple treatment as core material.

Material omitted as compared with the previous course:

- Permutations and Combinations: harder examples.

New material:

- Discrete probability: simple intuitive treatment.
- Statistics: weighted mean and (as core material) standard deviation.
- Difference equations.

OPTIONS

Note

The options contain material that is further along the path of mathematical development, but not specifically harder, than material in the core.

Further Calculus and Series

This contains classical material, much of which was in the previous Higher course — or, in the case of integration by parts, an earlier one (and hence familiar to teachers ... especially older ones!)

Material omitted as compared with the previous course:

- Some convergence tests.

New material:

- Integration by parts.
- Maclaurin series.

Further Probability and Statistics

This is effectively the same as the former option, apart from minor additions (for example: conditional probability, populations and samples) intended to be treated at a simple level. Hypothesis testing is more explicitly given a rationale. Since simple probability now appears in the core, more emphasis might be placed on the axioms as "capturing meaning".

Material omitted as compared with the previous course:

- Axioms of probability: proof of 5 theorems listed in the old course (but results still to be known for purposes of application).
- Standard scores: explicit treatment.

New material:

- Conditional probability.
- Normal distribution.
- Choice of measures of central tendency.
- Populations and samples.

Groups

Feedback on the option in the former course suggested that there was not quite enough

material to allow interesting treatment, and that the option should be either expanded or dropped. Since the topic is an archetypal representative of abstract algebra, and in addition is one in which some teachers have invested much work and enthusiasm, the option has been retained and expanded.

Material omitted as compared with the previous course:

- Examples of groups: symmetric difference.

New material:

- All later parts of option.

Further Geometry

This option builds on the new core material in transformation geometry. It includes the ellipse (a more biddable example of conic section than the parabola?); and it introduces the idea that theorems proved in one context (the circle) can be transferred to another (the ellipse) by transformations with suitable properties. Earlier work on isometries is developed, and extended to enlargements.

Material omitted as compared with the previous course:

- Parabola.

New material:

- Entire option.

THE ORDINARY COURSE

For a programme taken by a large part of the cohort, the Ordinary course has been rather specialised in style and content — catering particularly for the needs of those proceeding to further studies in technical, business and economic areas and in (for example) the biological sciences. Perhaps especially in view of national scientific and economic needs, there are good reasons for maintaining a course of this type. With the Ordinary Alternative course available for those who are not suited by the specialised work or the degree of abstraction which it involves, and with the better students hopefully moving to the Higher course, the new Ordinary course can therefore be geared to serve appropriately the students who wish to enter these areas, and can also provide a programme offering a non-trivial but not too great degree of abstraction to those who are able to avail of it.

The main needs as regards the style and content of the Ordinary course, therefore, appeared to be to enliven the course somewhat and perhaps to make it more immediately applicable. The style has thus been varied by the incorporation of topics with more everyday applications (probability, numerical methods and calculator use) and by the visual and psychomotor aspects of enlargements. Some material has been removed to make room for the new topics.

In view of the prospective "loss" of the more able students to the Higher course, the amount of difficult material is reduced by moving aspects of it to the options. These options therefore play a rather different role from those in the Higher course (the latter being further along the path of mathematical development rather than intrinsically more difficult). However, the Ordinary course options are intended to be rather short; they might occupy about three weeks teaching time, rather than the five weeks envisaged as a target figure for the Higher course units.

CORE

Arithmetic

This is seen as a "practical" part of the course, providing for students future lives in the world beyond school. It is intended that knowledge should be applied to everyday problems. Widespread use of calculators has made it important to encourage their appropriate use in school and to emphasise estimation and approximation.

New material:

- Estimation and approximation, relative error. Simpson's method, calculator use.
- Volume: right prism (? — old syllabus is not specific).

Algebra

The algebra has been gathered together and addressed more directly than in the previous syllabus. This has been done with the intention of focusing on algebraic skills, especially for those students proceeding to further study. However, it is not intended that the manipulations required should be unduly complicated.

Material omitted as compared with the previous course:

- Basics: Remainder Theorem (but see Factor Theorem).
- Equations: simultaneous equations in three unknowns.

Geometry

One aim of the geometry section of the course is to introduce the concept of proof: a "powerful idea" in our culture, and one which students proceeding from the Junior Certificate Ordinary course have yet to meet. It is addressed in the first topic below: and to allow time and headroom for emphasis on understanding, cuts are ruled out of this section. (They appear in one of the options.) The variety of geometrical presentations in the Junior Certificate syllabuses has caused some confusion: the presentation given here might eventually be part of a more unified approach.

The section on enlargements, which is common to the Ordinary and Ordinary Alternative courses, has three functions: it builds on the work on isometries done in the Junior Cycle; it provides a "drawing" — hence, spatial and psychomotor — element in the course; and it provides a shared topic for the two Ordinary level courses.

Material omitted as compared with the previous course:

- Synthetic geometry: various theorems; cuts (but see option).
- Co-ordinate geometry: tangents to circles other than centre (0,0); intersection of line and circle with centre not (0,0).

New material:

- Synthetic geometry: various theorems.
- Enlargements.

Trigonometry

The section has been shortened. In particular, radian measure is excluded, as it has no application in the course.

Material omitted as compared with the previous course:

- Radian measure.
- Drawing graphs of sine and cosine (recognition is still expected; see Functions).
- Simple identities.
- Compound angle formulae other than those named below.

Finite Sequences and Series

The concepts are important enough to merit introductory treatment in the core.

Material omitted as compared with the previous course:

- Limits of sequences and series (but see option).
- Binomial theorem (but see option).
- Applications.

Functions and Calculus

Functions and calculus constitute an important part of mathematical culture as well as a tool for those proceeding to further study.

Material omitted as compared with the previous course:

- Order relations.
- Differentiation: first principles for x.

Discrete Mathematics and Statistics

As indicated for the Higher course, this is an area of increasing importance in mathematics and also as regards its applications in everyday life. Much of the work on probability, in particular, is common to all three courses.

Material omitted as compared with the previous course:

- Statistics: median obtained from histogram.

New material:

- Simple permutations, and combinations. (explicitly mentioned).
- Probability.
- Statistics: interquartile range.

OPTIONS

Note:

The options in the Ordinary course consist basically of material withdrawn from the core of the former course. They are intended to be short but — by Ordinary course standards — quite advanced. Typically they involve problem-solving or modelling, and maybe other "powerful ideas".

Further Geometry

This option allows those who can bring cuts to life in the classroom an opportunity to do so.

New material:

- One new theorem.

Plane Vectors

See the rationale in the Higher course: the treatment here is the same.

Material omitted as compared with the previous course:

- Vectors: abstract approach based on a vector as a translation.

New material:

- Vectors treated intuitively.

Further Sequences and Series

This involves the "powerful idea" of a limit.

Material omitted as compared with the previous course:

- Binomial expansion.

Linear Programming

This topic was introduced in the sixties as being applicable, for example in the world of business.

Material omitted as compared with the previous course:

- Case in which two boundary lines are not the axes.

THE ORDINARY ALTERNATIVE COURSE

The Ordinary Alternative course is meant to offer a lively, relevant and mathematically meaningful education to those who do not require (or choose not to study) the rather "specialist" content of the Ordinary course. The mathematics involved is far from being trivial: parts of it — notably the development of parts of the number system — would not be out of place in the Higher course, though obviously the speed and level of treatment would be different. Thus, the Ordinary Alternative course is neither merely a maintenance course for social arithmetic nor a washed-out version of the Ordinary course. The serious heed paid to calculators, the recognition given to investigational and constructive learning, and the incorporation of strong visual and artistic elements make it genuinely complementary in scope and style.

Unfortunately, the lack of published teaching materials and the uncertainty as regards the acceptability and future of the course has militated against its full implementation. In particular, its unique flavour may have suffered. There remains a shortage of material in some

areas; but the sample and actual examination papers provide obvious targets, while some of the initially less familiar parts of the course (for instance, probability and enlargements) now appear in the textbooks for the Ordinary course and so are more accessible.

As the course does not replace a previous version, the syllabus listing that follows is not annotated as regards "ins and outs".

Number Systems
Material in this section is reviewed using a "new" approach, availing of the students' greater maturity and their (probable) reactions to regular use of the calculator.

Arithmetic
The emphasis is on everyday applications — but at a greater level of complexity than in the Junior Certificate Foundation course, with heed paid (say) to change in rates and charges. In section 1, the calculator is the object of study, as well as a tool for studying other topics.

Areas and Volumes
Attention is paid to both exact and approximate calculation. The latter is a topic of growing importance in mathematics because of the availability of information technology.

Algebra
As students without great power of abstraction tend to find algebra very hard, a very specific step-by-step approach is taken as regards building up some algebraic skills. Consideration is intended to be limited to the forms given in the syllabus.

Statistics and Probability
This area, highly relevant to everyday life, is common to all the courses.

Trigonometry
Introductory trigonometry, besides being intrinsically important, lends itself to practical work and applications.

Functions and Graphs
Emphasis is put on understanding the concept of a function (built up via carefully graded examples) and on being able to construct, read and interpret specified types of graphs.

Geometry
A variety of approaches is taken, several of them complementing the computational thrust of other parts of the syllabus. Hence they emphasise spatial development, visualisation and pattern as well as a knowledge of some of the fundamental results of Euclidean Geometry.

Investigations
The emphasis is on development of concepts and strategies for investigating mathematical problems and observing general patterns and results.