



Coimisiún na Scrúduithe Stáit
State Examinations Commission

Leaving Certificate Examination
Mathematics

Paper 1

Ordinary Level

2 hours 30 minutes

220 marks

Examination Number

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Day and Month of Birth

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For example, 3rd February
is entered as 0302

Centre Stamp

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Instructions

There are **two** sections in this examination paper.

Section A	Concepts and Skills	120 marks	6 questions
Section B	Contexts and Applications	100 marks	4 questions

Answer questions as follows:

- any **four** questions from Section A - Concepts and Skills
- any **two** questions from Section B - Contexts and Applications

Write your Examination Number in the box on the front cover.

Write your answers in blue or black pen. You may use pencil in graphs and diagrams only.

This examination booklet will be scanned and your work will be presented to an examiner on screen. Anything that you write outside of the answer areas may not be seen by the examiner.

Write all answers into this booklet. There is space for extra work at the back of the booklet. If you need to use it, label any extra work clearly with the question number and part.

The superintendent will give you a copy of the *Formulae and Tables* booklet. You must return it at the end of the examination. You are not allowed to bring your own copy into the examination.

You will lose marks if your solutions do not include relevant supporting work.

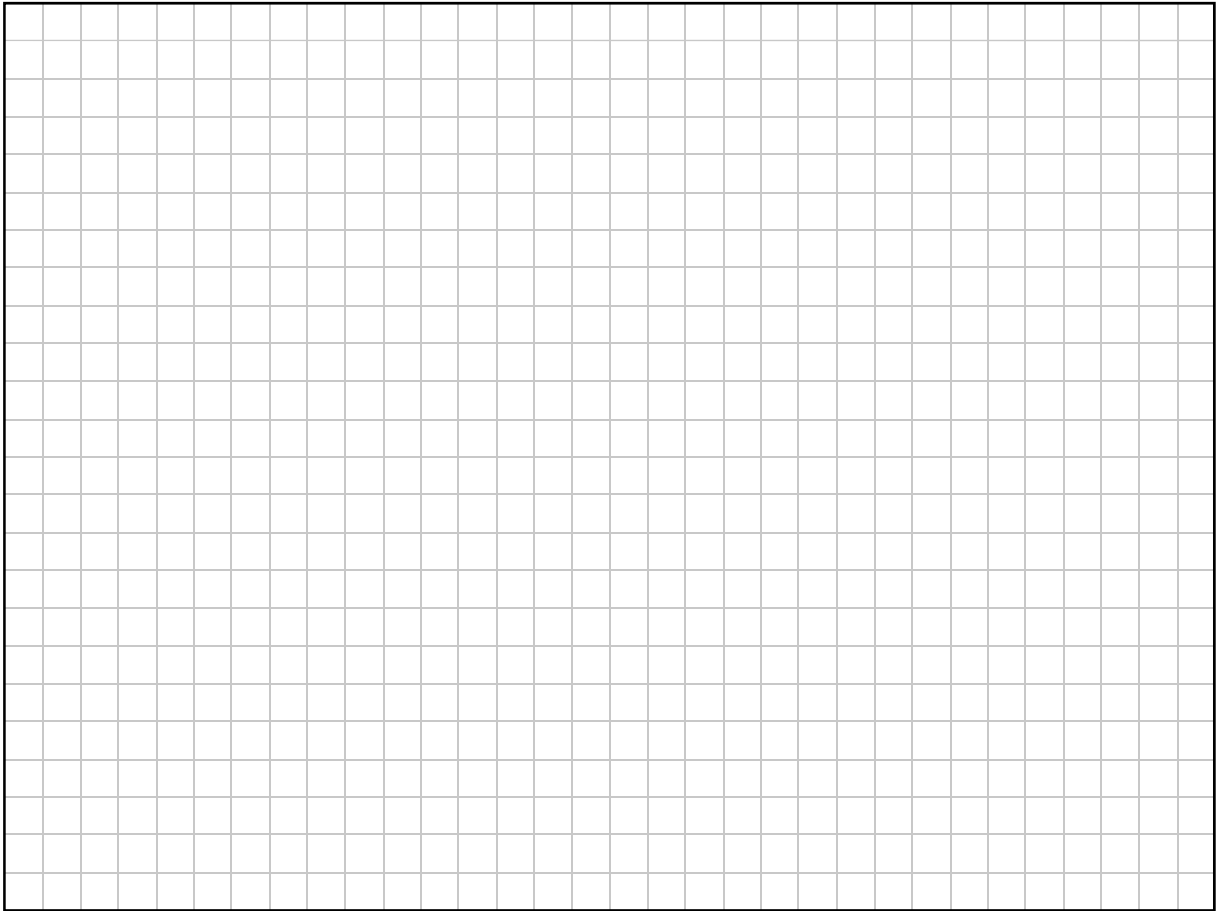
You may lose marks if you do not include appropriate units of measurement, where relevant.

You may lose marks if you do not give your answers in simplest form, where relevant.

Write the make and model of your calculator(s) here:

(c) Write the following as a single fraction in its simplest form:

$$\frac{2}{7x - 5} - \frac{4}{3x + 1}$$



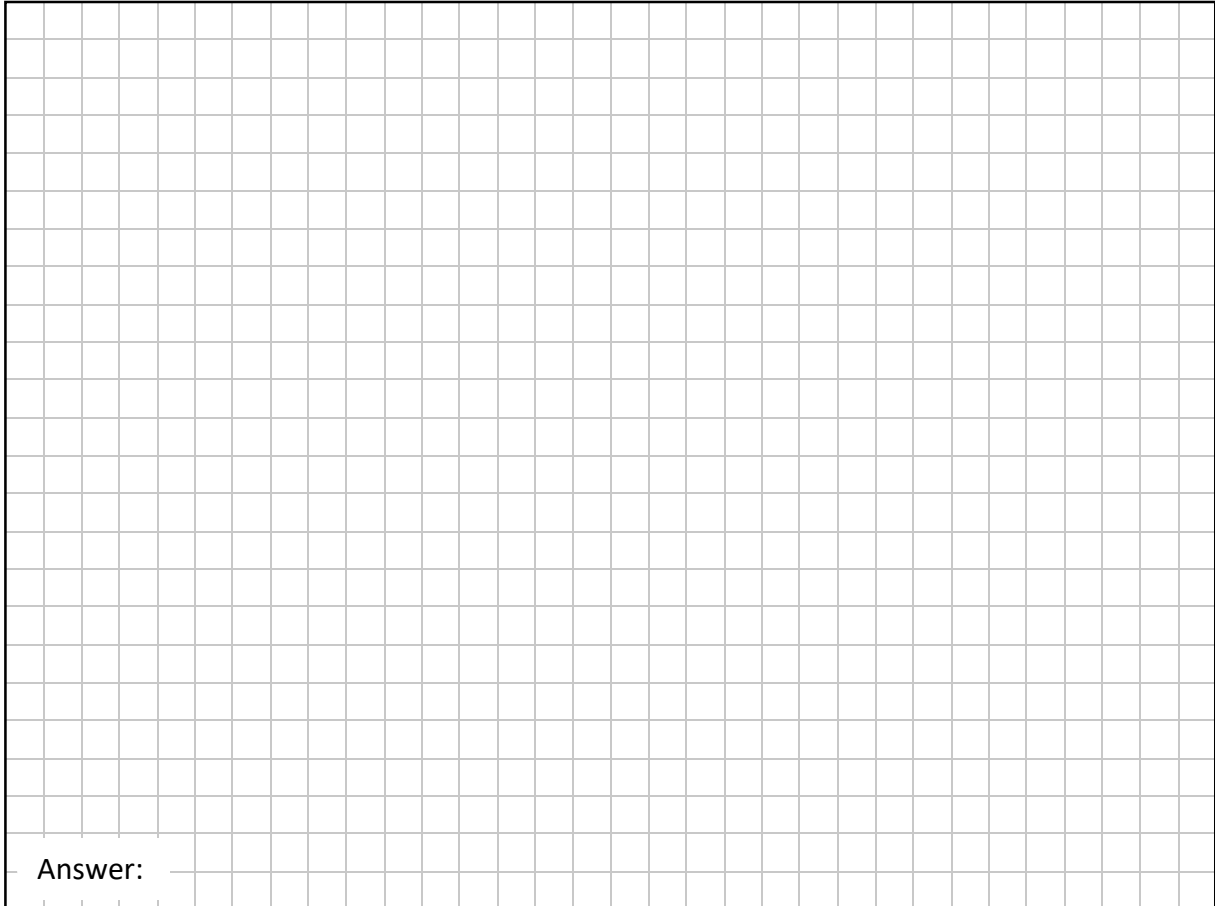
Question 2

(30 marks)

- (a) Gina buys a car for €22 000. It depreciates (decreases in value) by 20% each year.
(i) Work out how much it will be worth exactly 2 years after Gina buys it.

- (ii) On 1 January 2025, the value of this car will be €10 000.
Work out its value exactly 1 year earlier, on 1 January 2024.

- (b) Two students are looking on the internet to find the value of the speed of light. One website gives it as 3×10^8 metres per second. Another says that light travels approximately 9.5 trillion kilometres in a year. Given that a trillion is a million million, find out whether or not these two answers are consistent with each other.

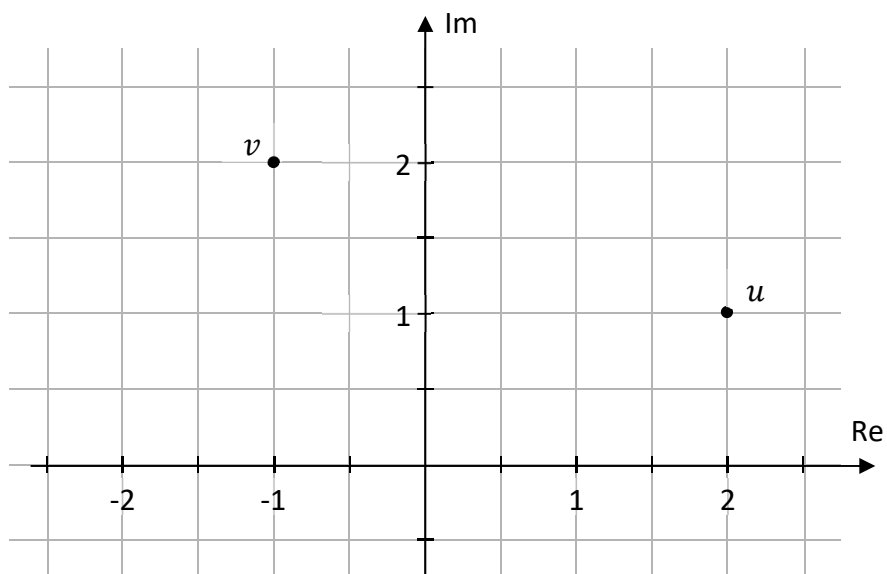


Answer: _____

Question 3

(30 marks)

Two complex numbers, u and v , are shown on the Argand diagram below.

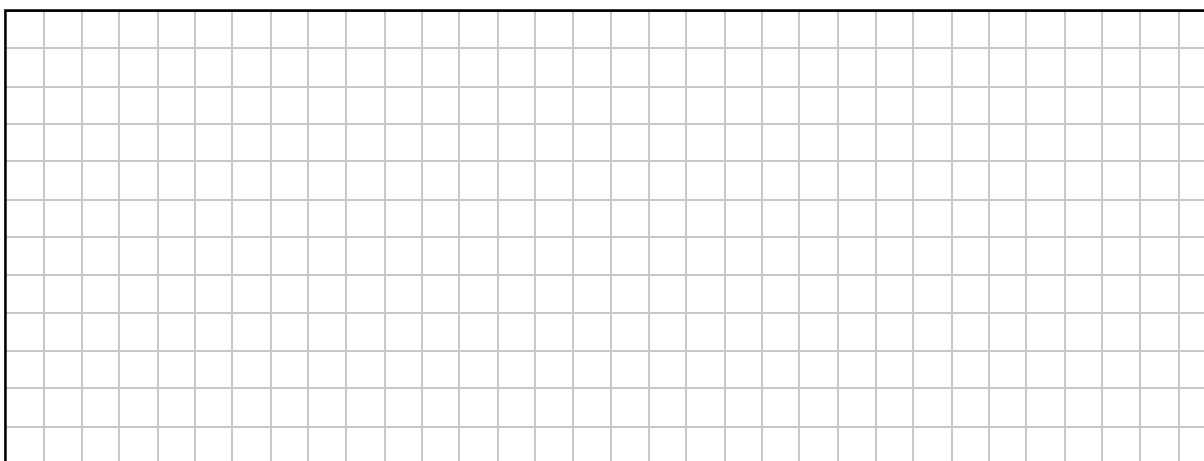


- (a) Write u and v in the form $a + bi$, where $a, b \in \mathbb{R}$ and $i^2 = -1$.

$u =$

$v =$

- (b) (i) Verify, by calculation, that $|u| = |v|$. Show your working out.



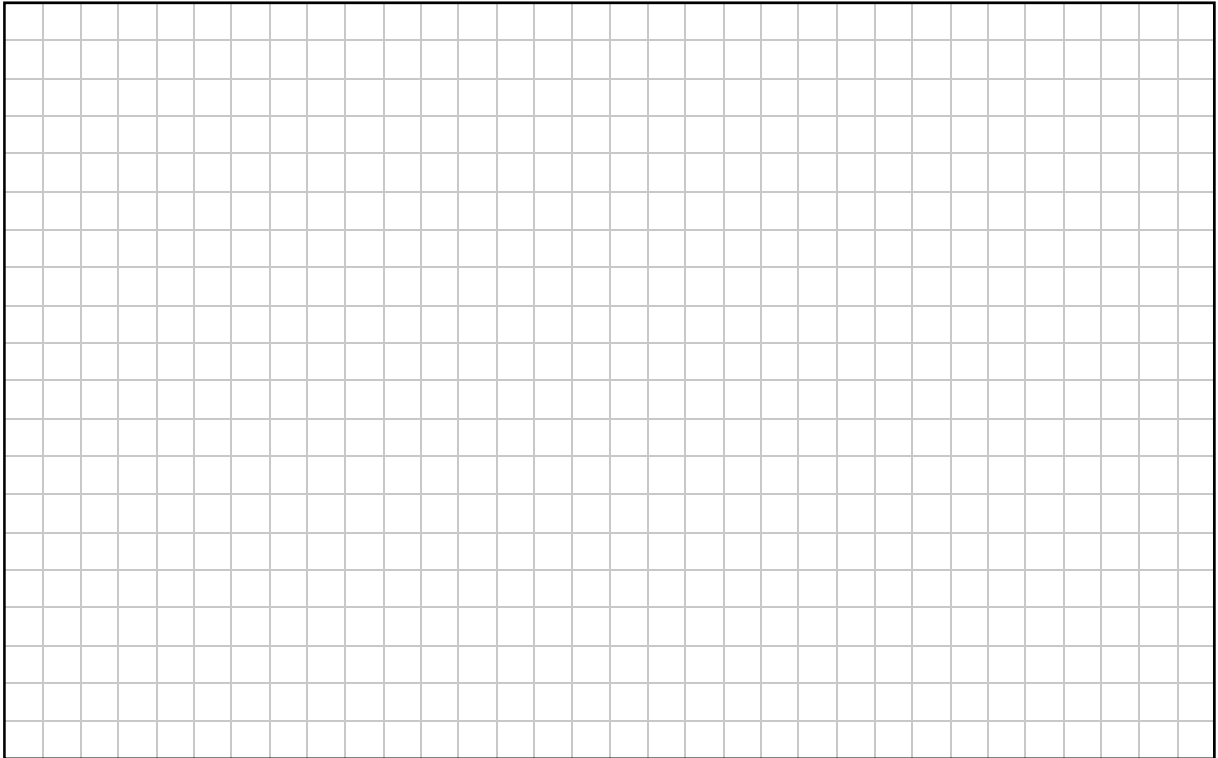
- (ii) Draw a complex number w on the diagram above, so that $|w| = |u| = |v|$, and write w in the form $a + bi$, where $a, b \in \mathbb{R}$ and $i^2 = -1$.

$w =$

(c) z is a complex number, so that $u \times z = v$.

Find z .

Give your answer in the form $a + bi$, where $a, b \in \mathbb{R}$ and $i^2 = -1$.

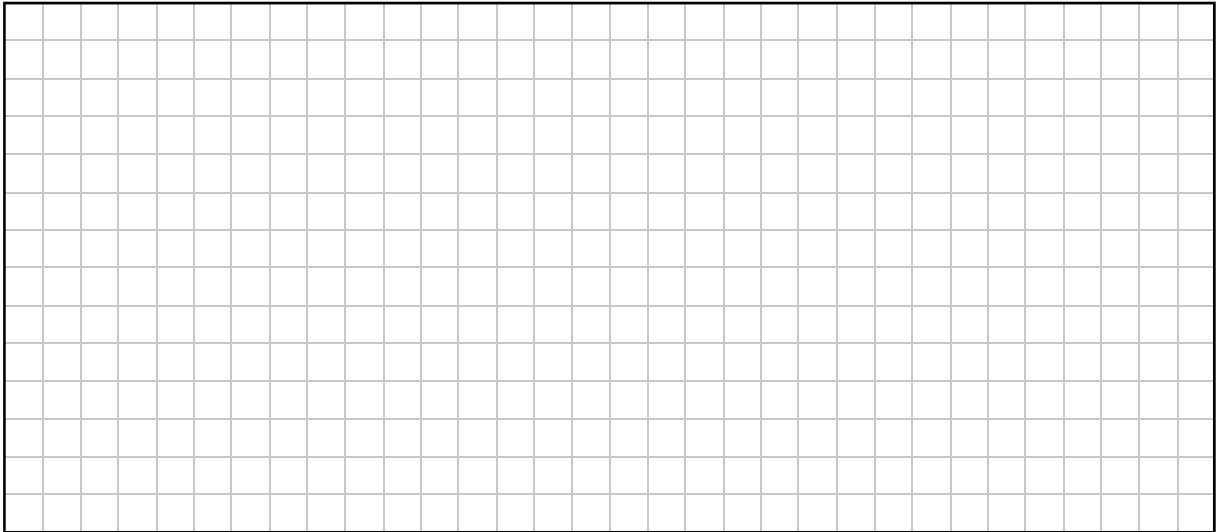


Question 4

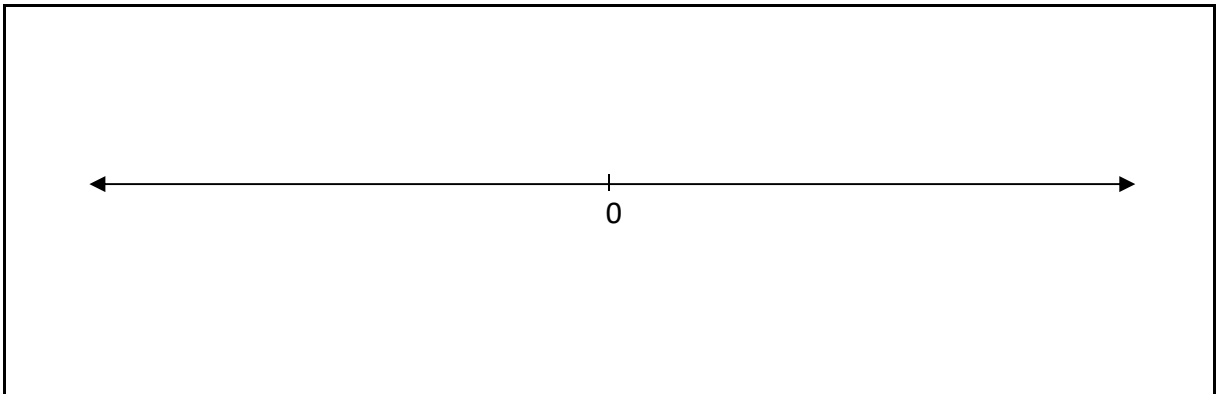
(30 marks)

(a) (i) Solve the following inequality, for $x \in \mathbb{R}$:

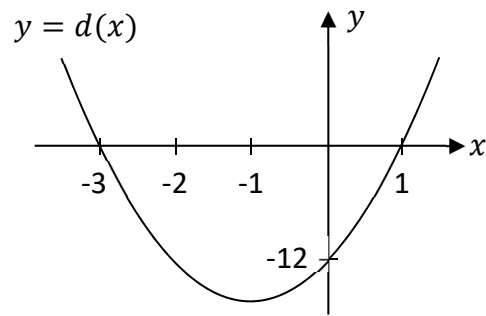
$$5 - 2x \leq 11$$



(ii) Show the solution set on the number line below. Remember that $x \in \mathbb{R}$.



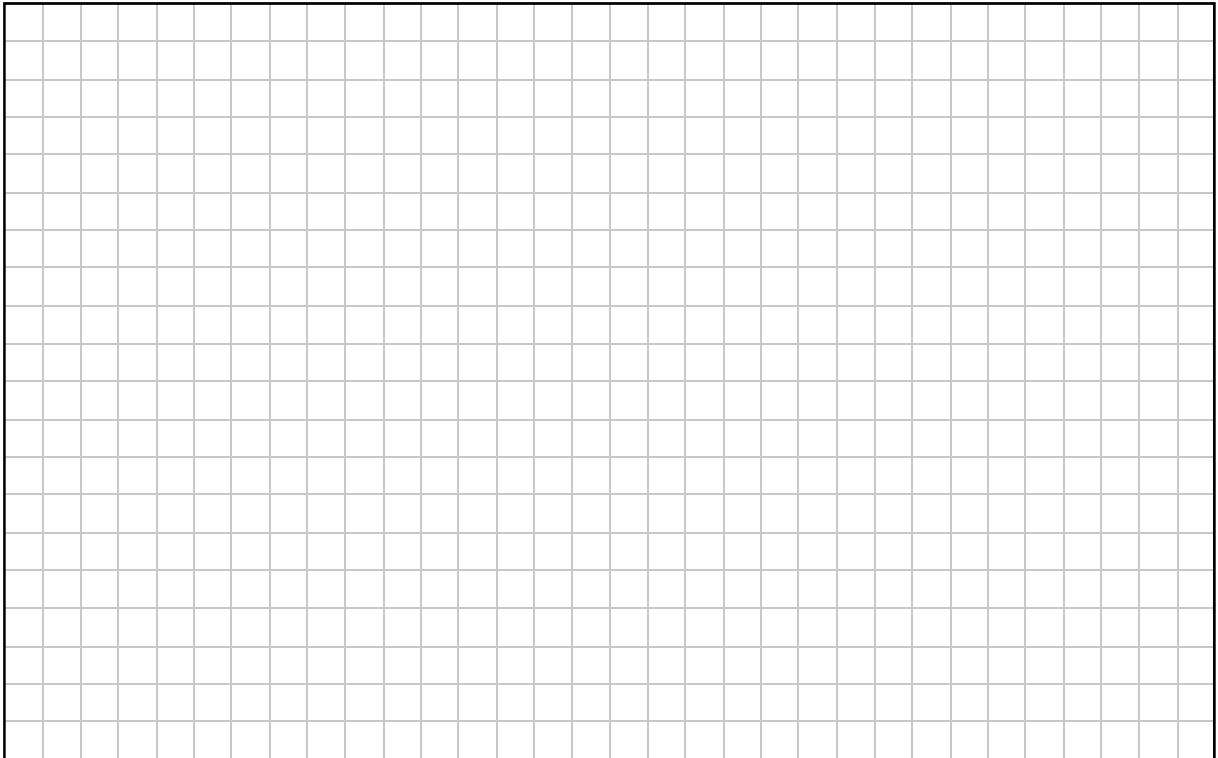
(b) The graph of the quadratic function $d(x)$ is shown on the co-ordinate diagram below.



(i) Use the graph to write down the two solutions of the equation $d(x) = 0$.

$x =$ and $x =$

(ii) Hence, write $d(x)$ in the form $d(x) = ax^2 + bx - 12$, where $a, b \in \mathbb{R}$.



Question 5

(30 marks)

(a) Let $f(x) = 2x^3 - 3x^2 + 7x$, where $x \in \mathbb{R}$.

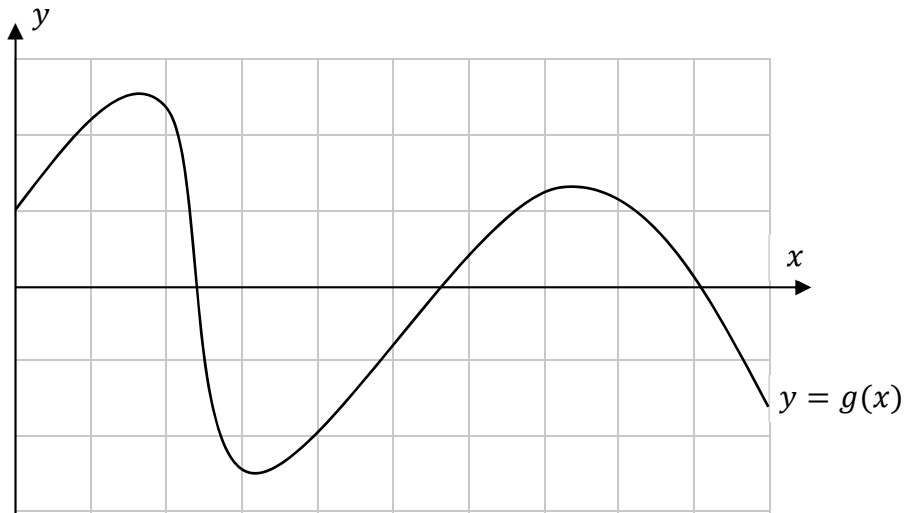
(i) Find $f'(x)$, the derivative of f .

(ii) Find the value of $f'(x)$ when $x = 1$.

(iii) Find the equation of the tangent to the curve $y = f(x)$ at $x = 1$.

(b) The graph of $y = g(x)$ is shown on the co-ordinate diagram below, for a particular range of $x \in \mathbb{R}$.

On the graph, **mark** the three points for which $g'(x) = 0$, where $g'(x)$ is the derivative of g . **Label** these points **A**, **B**, and **C**.



Question 6**(30 marks)**

The functions h and p are defined for $x \in \mathbb{R}$ as follows:

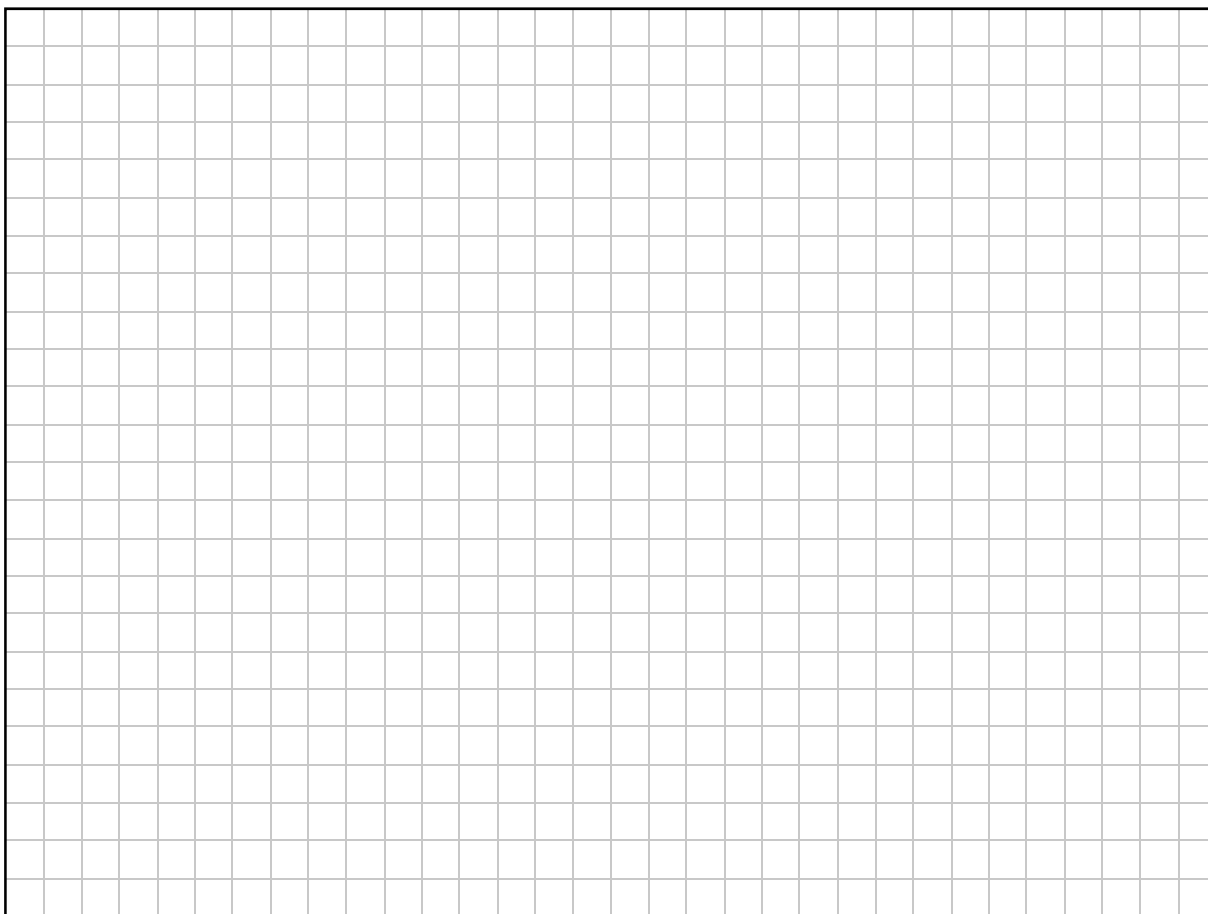
$$h : x \mapsto 2x - 3$$

$$p : x \mapsto x^2 - 1$$

(a) Find $p(5)$.

(b) (i) Show that $p(h(x)) = 4x^2 - 12x + 8$.

- (ii) Solve $p(h(x)) = 4$.
Give each answer correct to 2 decimal places.



Answer **any two** questions from this section.

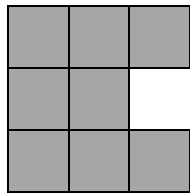
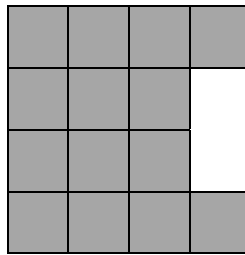
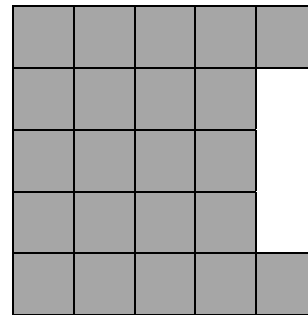
Question 7**(50 marks)**

Darragh is laying square tiles to make a large patio.

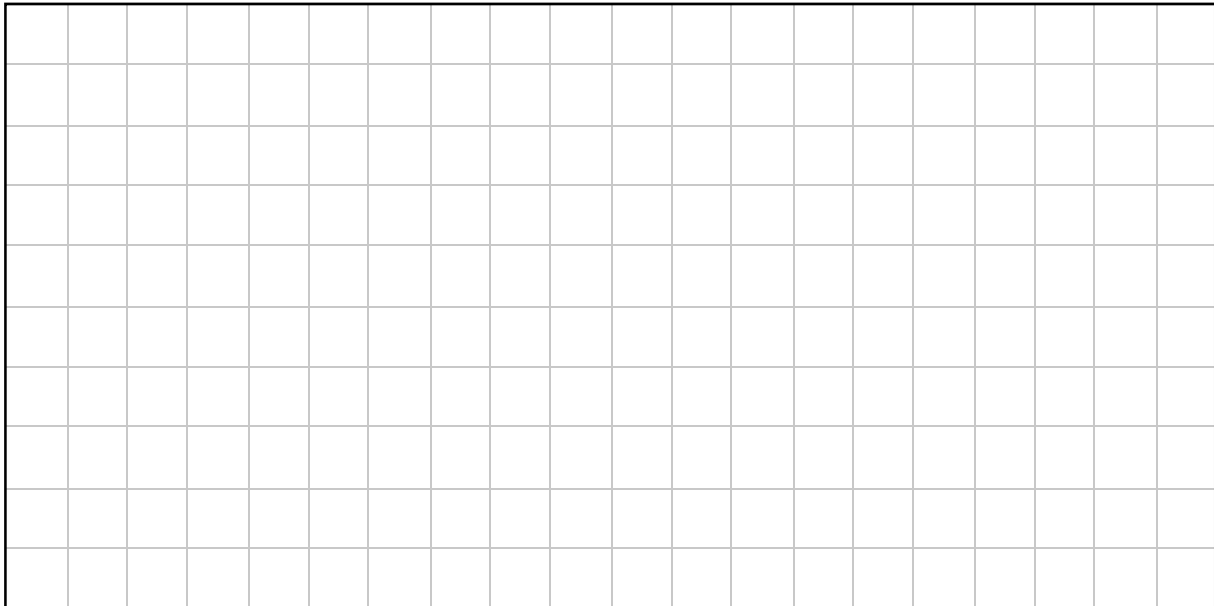
He draws a diagram at the end of each day to record his progress.

The diagrams below show the tiles that have been laid up to the end of each of the first 3 days.

Darragh expects to follow the pattern shown below for several weeks in order to complete the work.

**Day 1****Day 2****Day 3**

- (a) Using the pattern above, draw a diagram to show the tiles that have been laid up to the end of Day 4.



- (b) Complete the table below to show the **total** number of square tiles that have been laid up to the end of each of the first 7 days. The first two have already been done for you.

Day	1	2	3	4	5	6	7
Total number of square tiles used	8	14					

- (c) Darragh works out that T_n , the total number of tiles laid up to the end of day n , is given by the following formula, as long as Darragh is still laying tiles on day $n \in \mathbb{N}$:

$$T_n = (n + 2)^2 - n$$

- (i) Show that this formula gives the correct value for the total number of tiles that have been laid up to the end of Day 2.

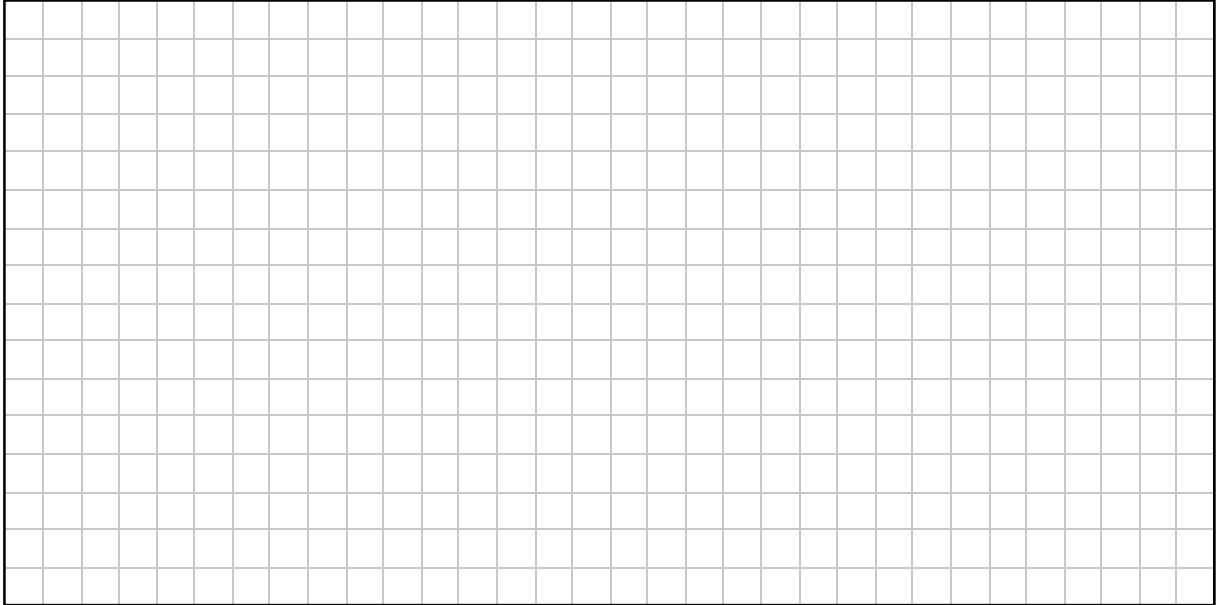
- (ii) Write the formula for T_n above in the form:

$$T_n = an^2 + bn + c$$

where $a, b, c \in \mathbb{Z}$.

This question continues on the next page.

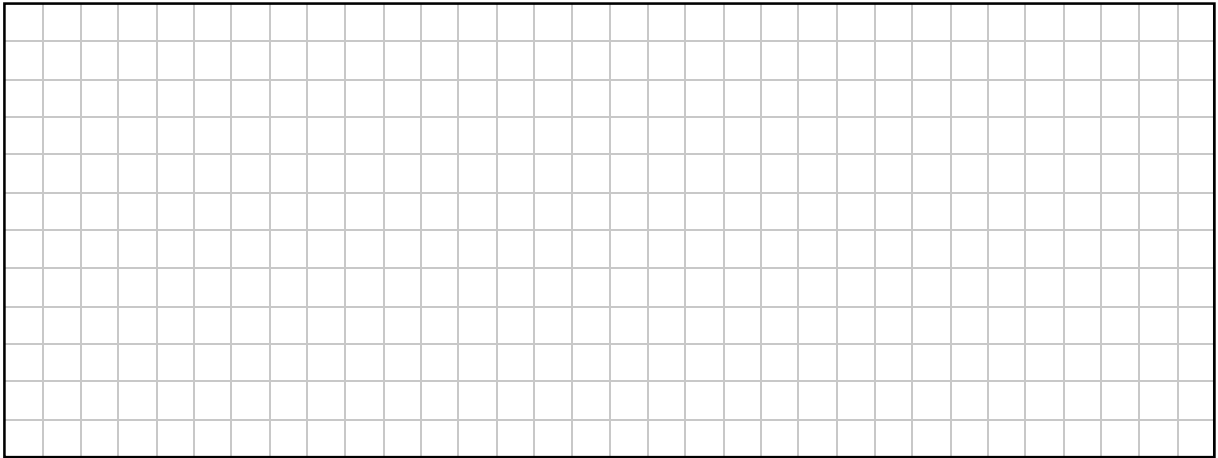
(d) Work out how many tiles Darragh laid on the 15th day.



(e) When the patio is completed, it will be made up of 554 square tiles.

(i) Each square tile measures 60 cm by 60 cm.

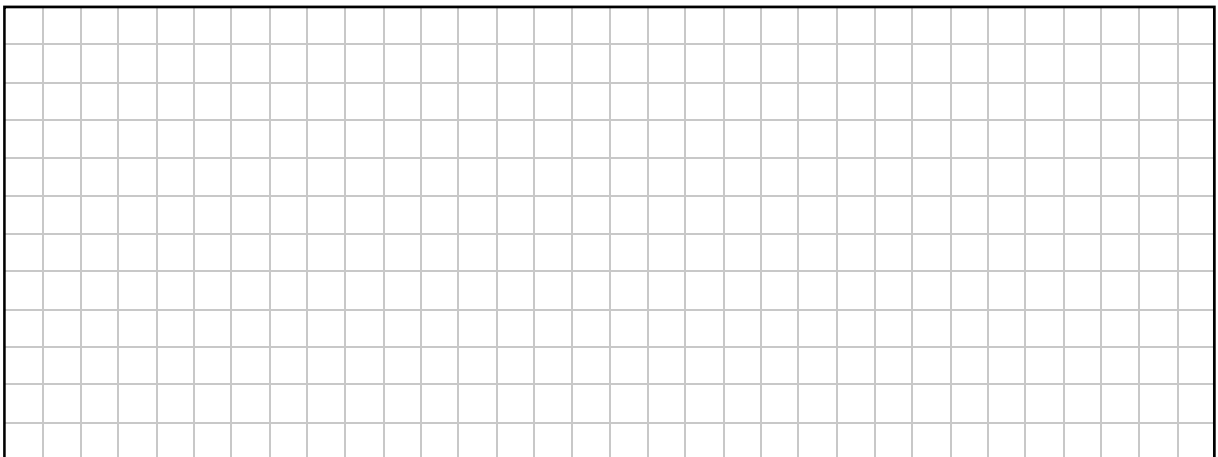
Work out the **total area** of the patio, in m^2 , once it is completed.



(ii) Darragh charges €700 for each 20 m^2 of patio, plus VAT at 23%.

Work out how much Darragh charges, in total, for the patio.

Give your answer correct to the nearest euro.



Question 8

(50 marks)

Gina is on the roof of a building and Harry is at the bottom of the building.

Harry throws a ball upwards. The height of his ball above the ground during its flight is given by $h(t)$, where $t, h \in \mathbb{R}$, t is in seconds, and h is in metres.

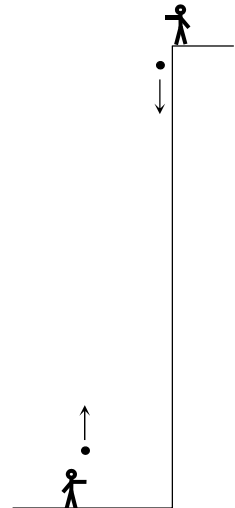
Harry throws his ball upwards when $t = 0$.

The graph of $y = h(t)$ is shown on the next page.

When $t = 1$, Gina throws her ball downwards. The balls do not collide.

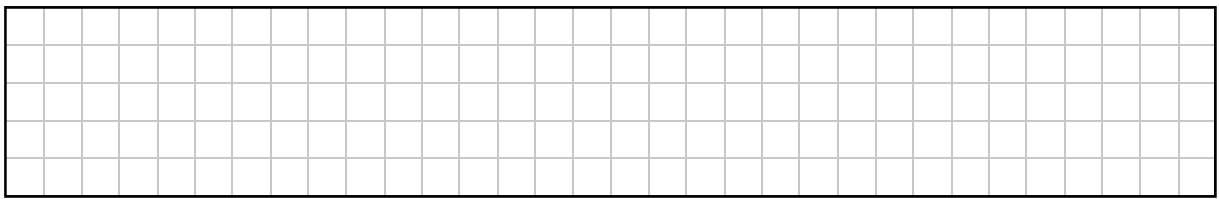
The height of Gina's ball above the ground during its fall is given by $g(t)$, where $t \geq 1$ and $g \in \mathbb{R}$ is in metres:

$$g(t) = 18 + 10t - 5t^2$$

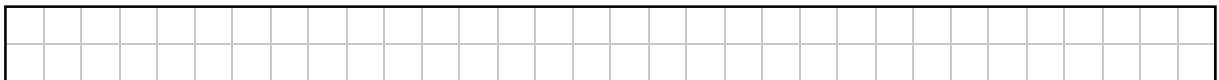


- (a) (i) Complete the table below to show the values of $g(t)$ for the given values of t .

t	1	1.5	2	2.5	3	3.1
$g(t)$		21.75				0.95



- (ii) Draw the graph of $y = g(t)$ on the co-ordinate diagram on the next page, using the same axes and scales. **Note:** $t \geq 1$ for $g(t)$.



- (b) Use the graphs of $h(t)$ and $g(t)$ on the next page to estimate the answer to each of the following questions. In each case, **show your work on the diagram** on the next page.

- (i) How much higher than Harry's ball is Gina's ball, half a second after Gina drops her ball (at $t = 1.5$)?

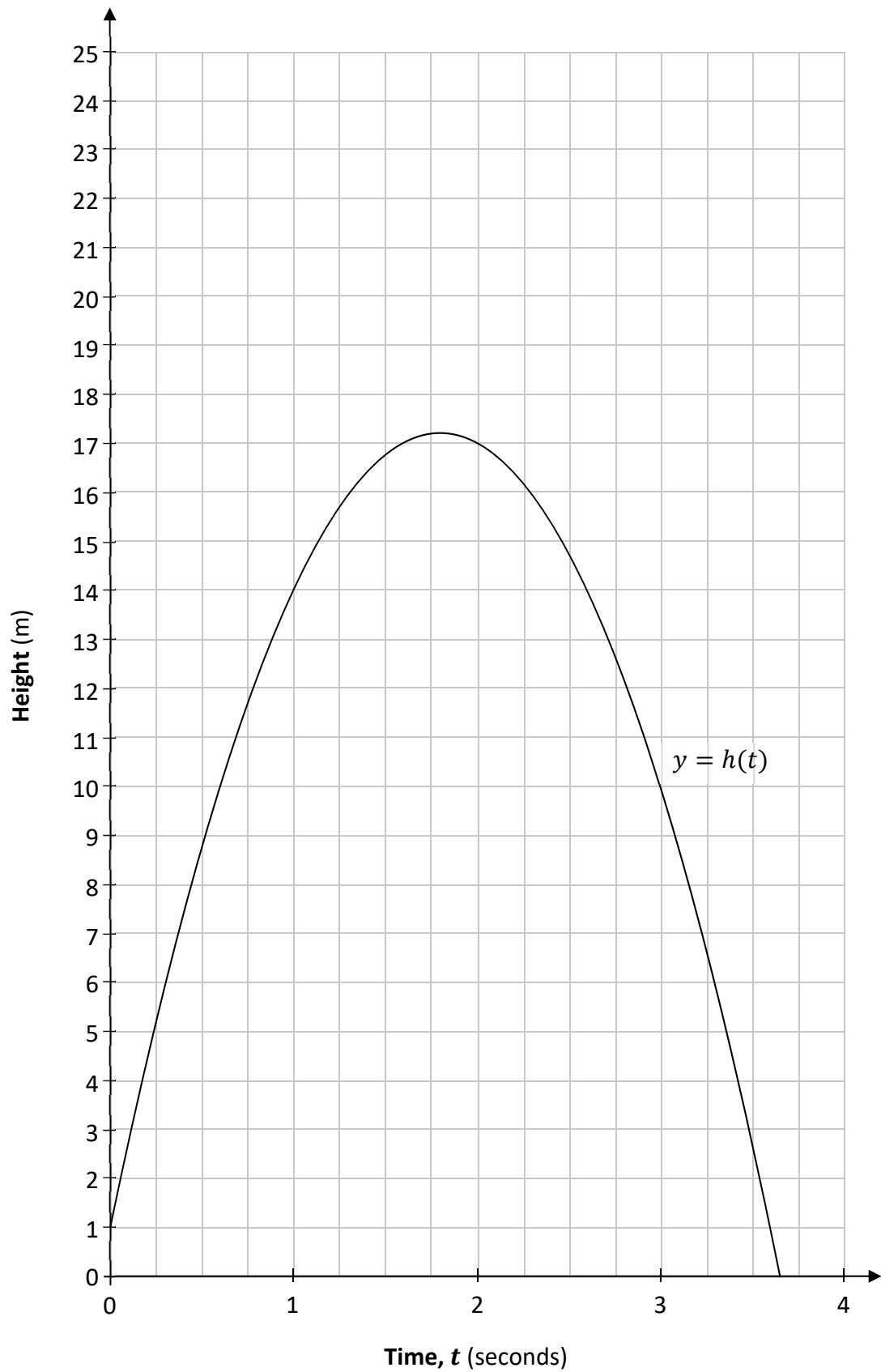
Answer:

- (ii) How long does it take Harry's ball to reach its maximum height?

Answer:

- (iii) At what height does Gina's ball pass Harry's ball?

Answer:



This question continues on the next page.

(c) The height of Harry's ball during its fall, $h(t)$, is given by:

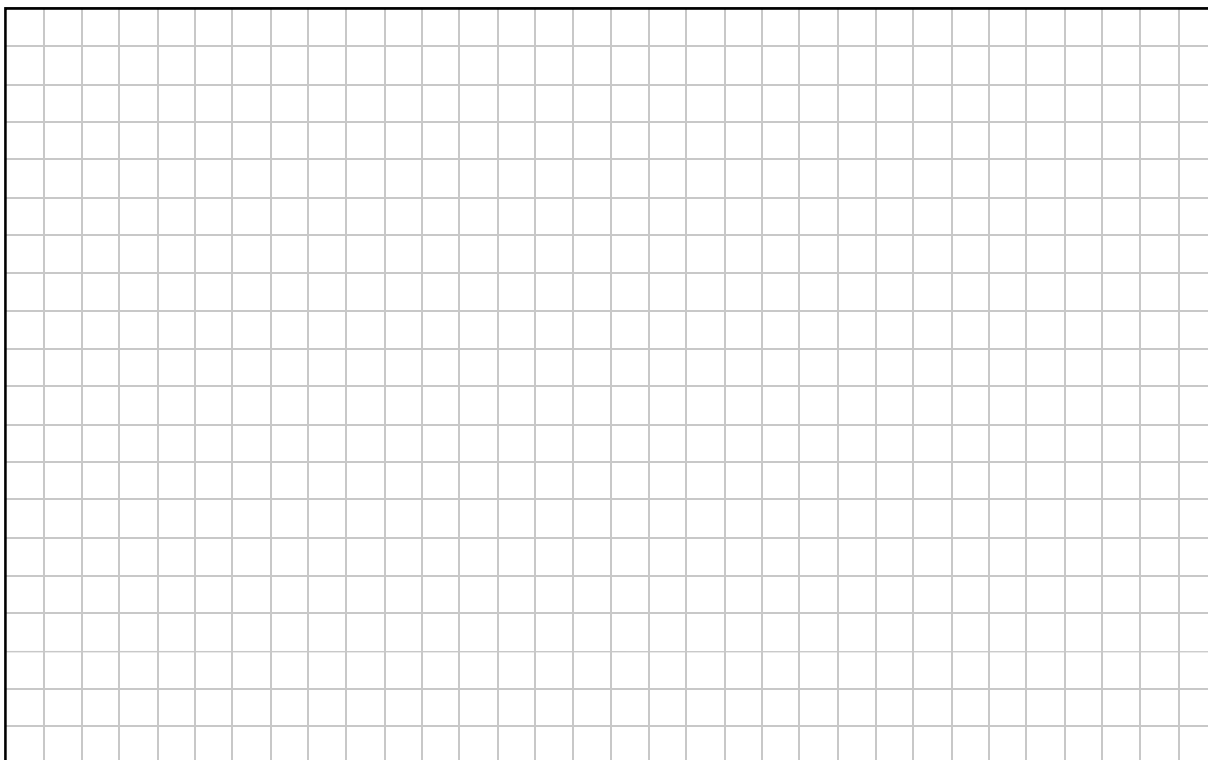
$$h(t) = 1 + 18t - 5t^2$$

Use this, and the fact that $g(t) = 18 + 10t - 5t^2$, to answer the following questions. These are the same questions as in **part (b)**, but here you will need to use algebra to answer them.

(i) How much higher than Harry's ball is Gina's ball, half a second after Gina drops her ball ($t = 1.5$)? Show your working out.

(ii) Use calculus to find how long it takes Harry's ball to reach its maximum height.

- (iii) By solving an equation, find the **height** at which Gina's ball passes Harry's ball.
Give your answer correct to 1 decimal place.



Question 9

(50 marks)

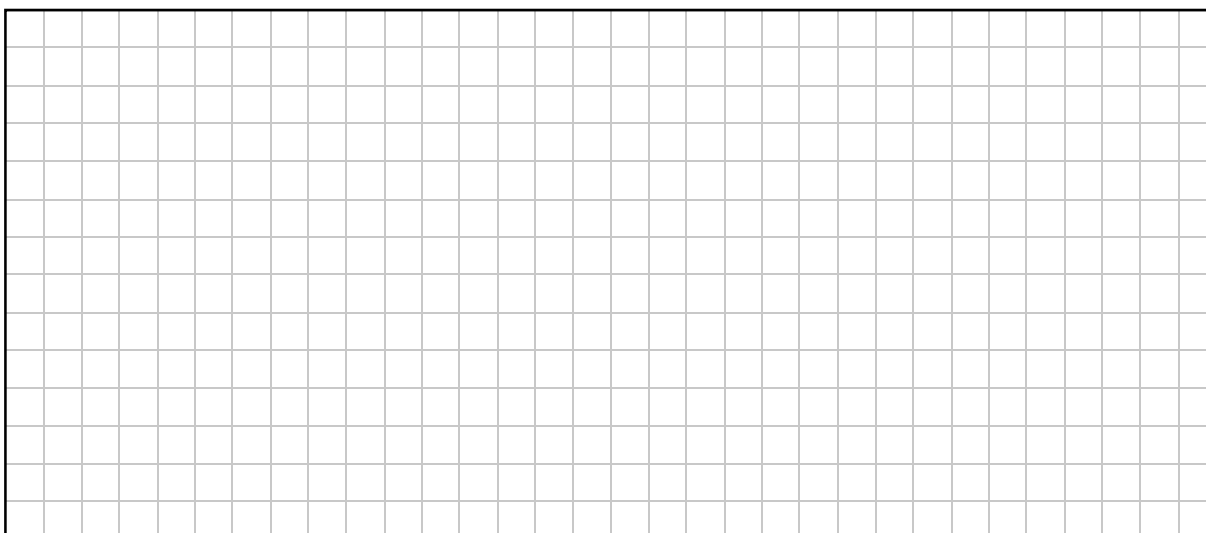
Mary is saving money.

She starts saving with 136 coins, worth a total of €51.

Each week, Mary’s mother gives her €3 to add to her savings, as follows:
she gives Mary one €1 coin, two 50-cent coins, and five 20-cent coins.

- (a)** Use this information to complete the table below, showing the total number of coins and the total amount of money that Mary has saved as the weeks pass.
Each answer in the **last row** should be an expression in terms of n .

	Total number of coins	Total amount of money (€)
Start	136	51
After 1 week		
After 2 weeks		
After 3 weeks		
After 4 weeks		
After n weeks ($n \in \mathbb{N}$)		



- (b) Will there ever be a time at which the total number of coins that Mary has is equal to the total amount of money (in euro) that she has saved?

If so, find when that happens; if not, explain why not.

- (c) Mary keeps her money in three different money boxes.
Box 1 has only €1 coins, Box 2 has only 50-cent coins, and Box 3 has only 20-cent coins.
Each box contains the same amount of money.

Mary starts with 136 coins in total in her three money boxes, worth a total of €51.

Work out the number of coins in each money box at this time.

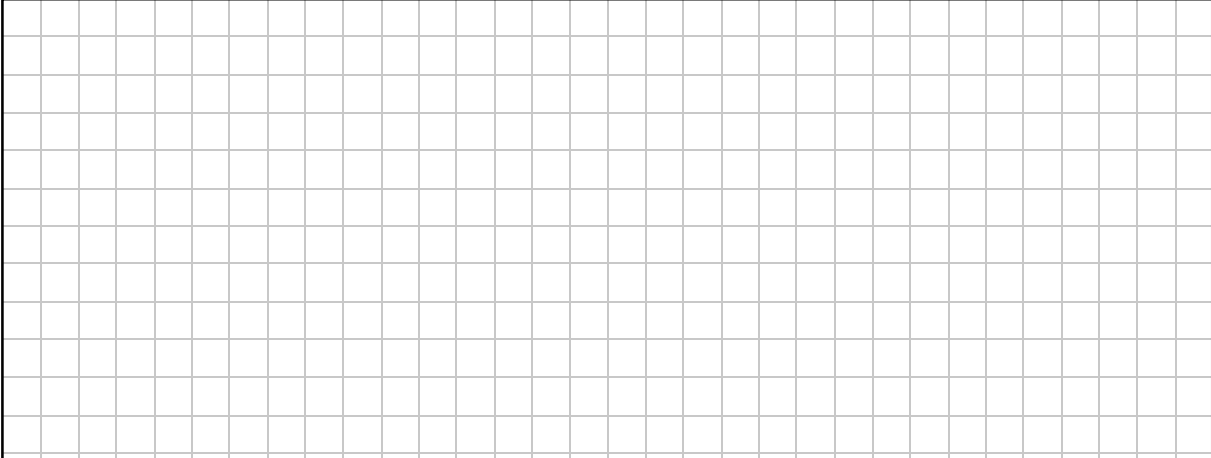
Money box	Box 1	Box 2	Box 3
Number of coins in this money box			

This question continues on the next page.

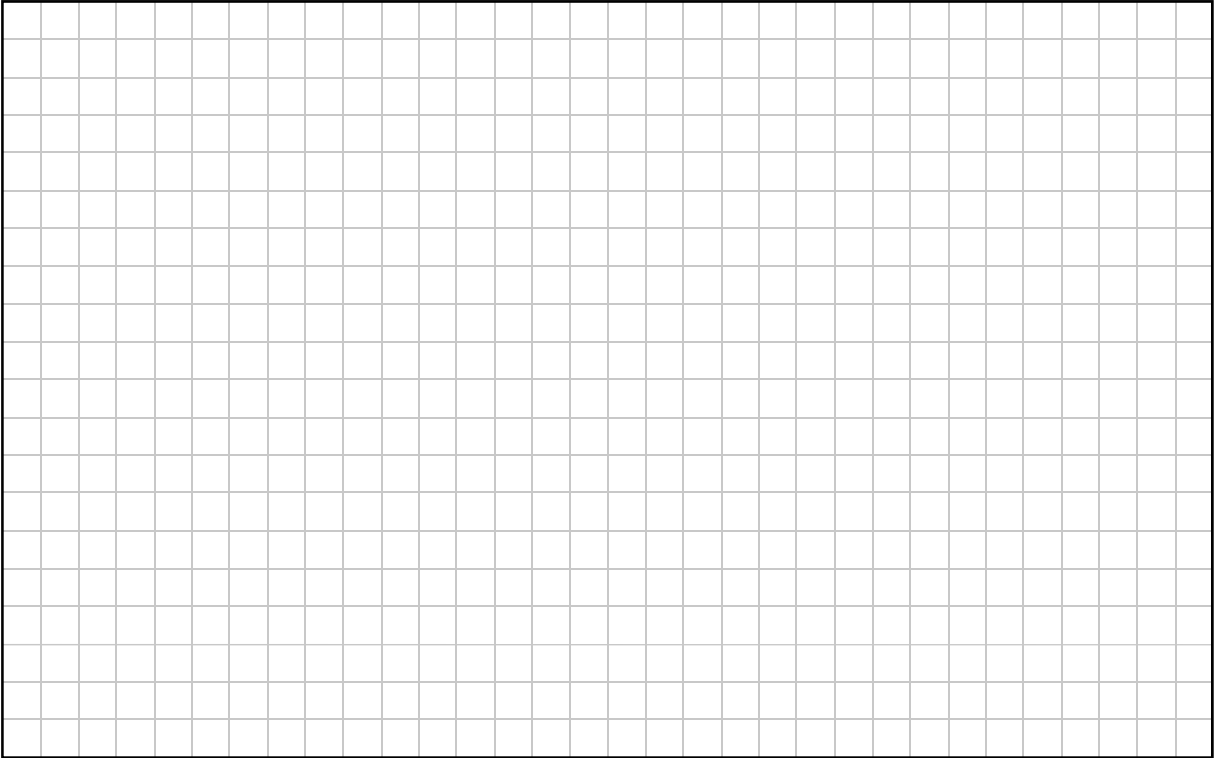
(d) Mary's mother pays USC at each of the three rates shown in the following table:

Band	Rate
First €12 012	0.5%
€12 012 to €20 484	2%
Above €20 484	4.5%

(i) Work out the USC that Mary's mother pays at the 0.5% rate **and** the 2% rate.

			
		USC at 0.5% rate: _____	USC at 2% rate: _____

- (ii) Mary's mother pays a total of €822.72 in USC.
Work out her gross income.



Question 10

(50 marks)

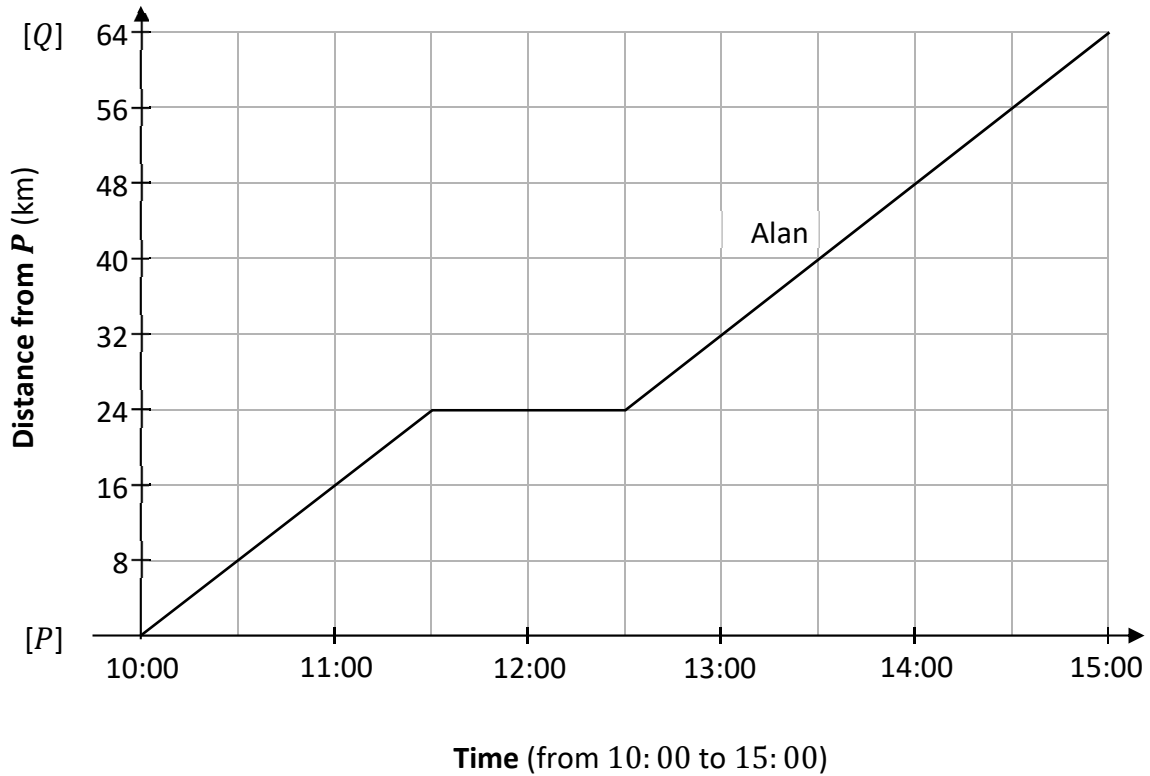
Two friends, Alan and Brendan, separately cycled a 64 km journey from P to Q .
A third friend, Colin, cycled from Q to P on the same day.

The diagram below shows the graph of Alan's journey.

Time is measured in hours, starting at time 10:00.

Distance is measured in kilometres from P , along the route travelled.

Alan left P at 10:00, cycled at a constant speed, stopped for an hour for a break, and then continued at the same speed to Q . He arrived at Q at 15:00.



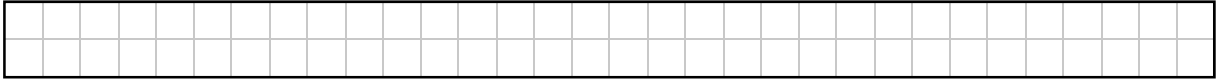
(a) At what time of the day did Alan stop for a break?

Answer:

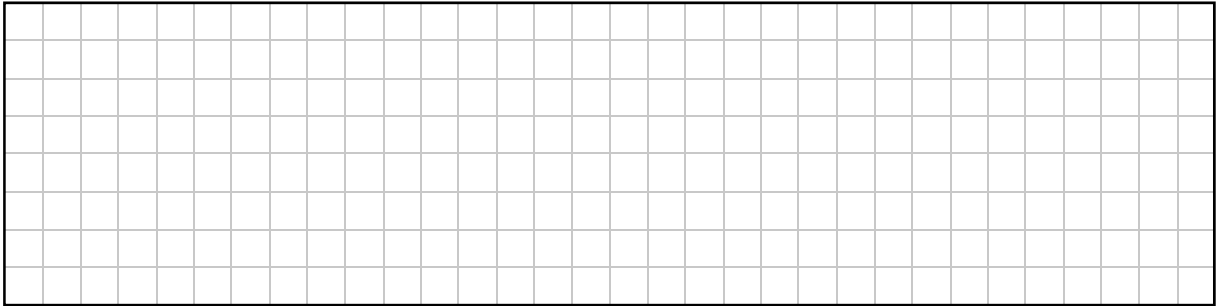
(b) Work out Alan's constant speed when he was cycling.

- (c) Brendan left P at 12:00.
He cycled at a constant speed, without stopping, until he reached Q .
He passed Alan at 14:00.

- (i) **Draw** the graph of Brendan's journey on the diagram on the previous page, using the same axes and scales. Label the graph clearly.

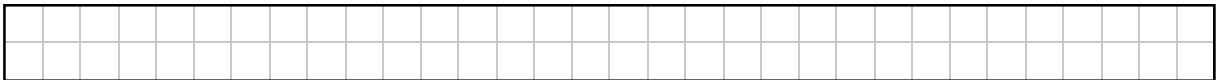


- (ii) Work out Brendan's constant speed when he was cycling.

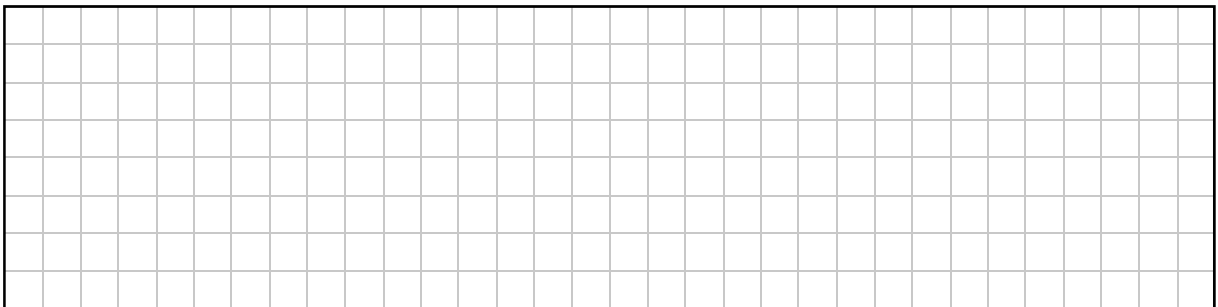


- (d) Colin cycled in the **opposite** direction, from Q to P along the same road.
Colin left Q at 10:00 and cycled at a constant speed, without stopping, until he reached P .
He arrived at P at 15:00.

- (i) **Draw** the graph of Colin's journey on the diagram on the previous page, using the same axes and scales. Label the graph clearly.

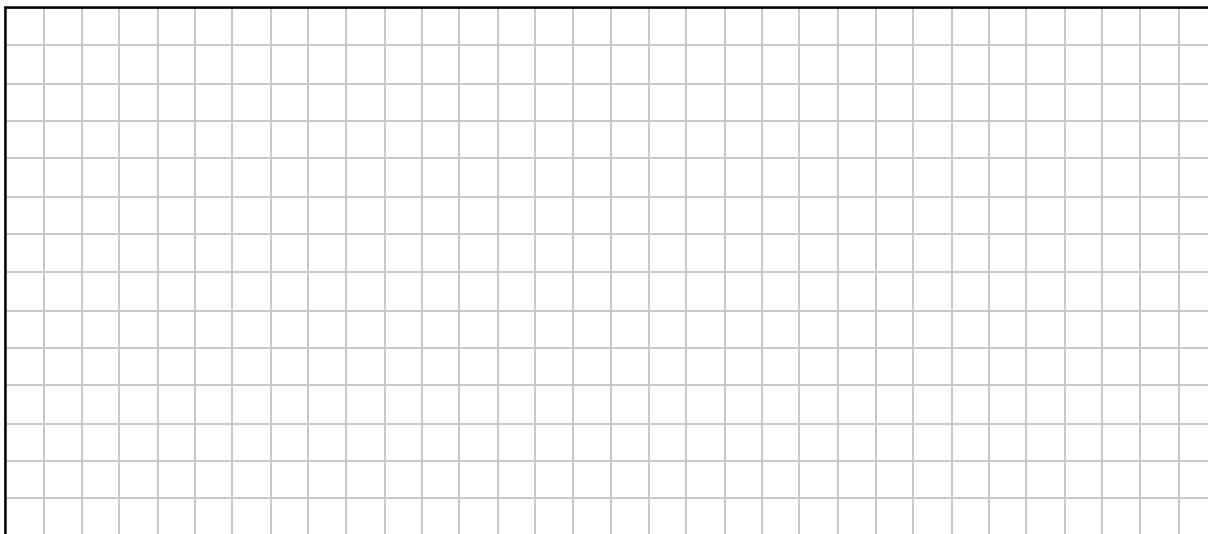


- (ii) Use your graphs to estimate the time of day at which Colin and Brendan passed each other.



This question continues on the next page.

- (e) Write a formula to represent Colin's distance from P at any given time during his journey. Your formula should be written in terms of t , where $t \in \mathbb{R}$ is the time in hours from 10:00. Remember that Colin left Q at 10:00 and arrived at P , 64 km away, at 15:00.



- (f) On a different day, Alan cycles in a race. This time, his speed changes throughout the race. The distance, in km, he has travelled from the starting line during the race is given by:

$$d(t) = 6t^3 - 12t^2 + 40t$$

where t is the time in **hours** from the start of the race, and $t \in \mathbb{R}$, $0 \leq t \leq 1$.

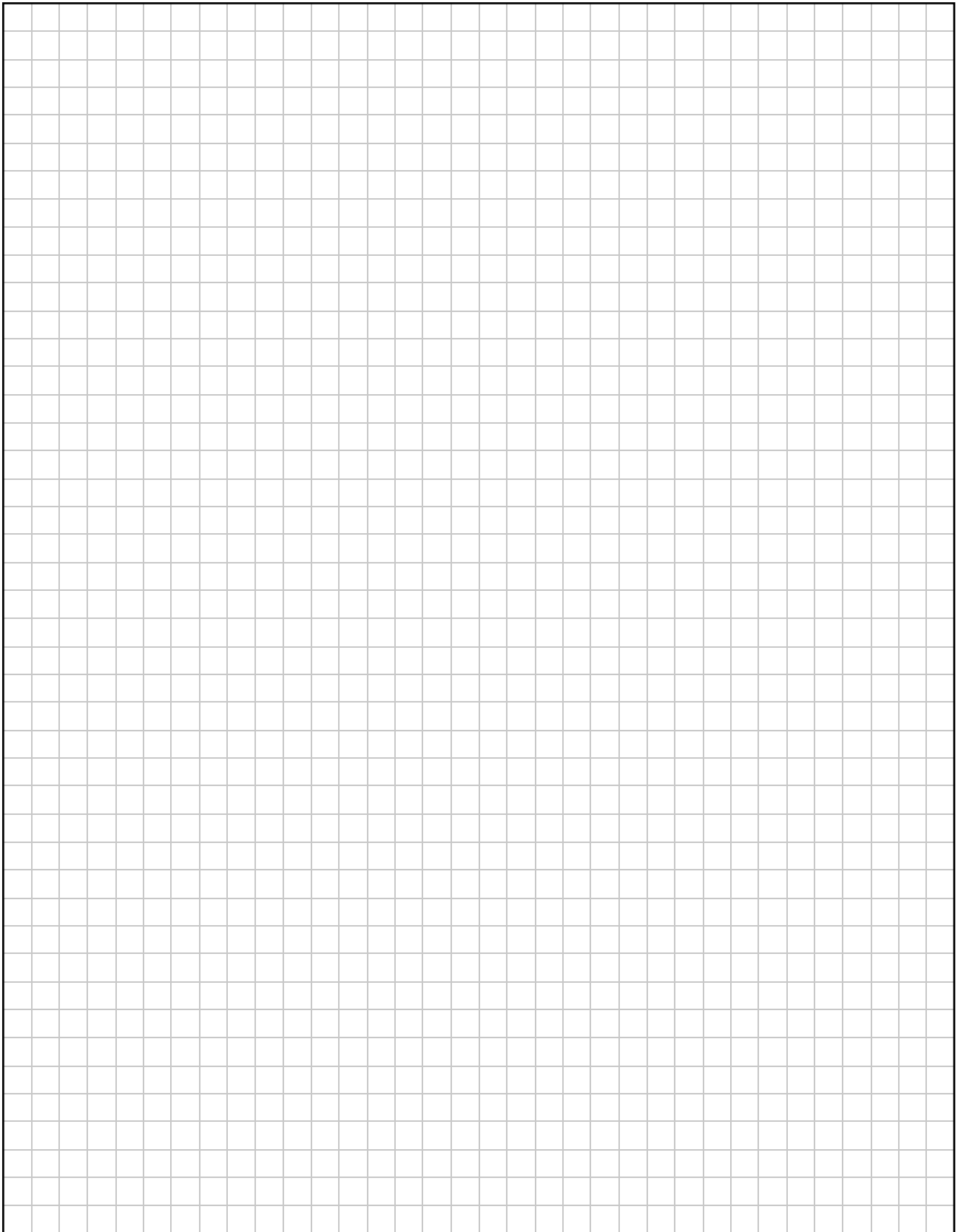
Alan's **speed** during the race is given by $d'(t)$, the derivative of $d(t)$.

Use this to find Alan's speed 30 **minutes** after the start of the race.



Page for extra work.

Label any extra work clearly with the question number and part.



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Leaving Certificate – Ordinary Level

Mathematics Paper 1

2 hours 30 minutes