



**Coimisiún na Scrúduithe Stáit
State Examinations Commission**

LEAVING CERTIFICATE EXAMINATION, 2009

MATHEMATICS – ORDINARY LEVEL

PAPER 1 (300 marks)

FRIDAY, 5 JUNE – MORNING, 9:30 to 12:00

Attempt **SIX QUESTIONS** (50 marks each).

WARNING: Marks will be lost if all necessary work is not clearly shown.

**Answers should include the appropriate units of measurement,
where relevant.**

1. (a) Conor and Alice share 50 apples in the ratio 3 : 7.
- (i) How many apples does Conor get?
 - (ii) How many apples does Alice get?
- (b) Barbara works 35 hours a week and she is paid €12·60 per hour.
- (i) Find her total weekly pay.
 - (ii) Barbara pays tax at the rate of 20% on all her income and has weekly tax credits of €53. Calculate her weekly take-home pay.
 - (iii) In one particular week, Barbara worked 4 additional hours at the same rate of pay. By how much did her take-home pay increase that week?
- (c) €7500 was invested for 2 years at $r\%$ per annum compound interest.
- (i) The amount of the investment at the end of the first year was €7860. Find the value of r .
 - (ii) At the start of the second year € X was withdrawn from the account. The interest earned during the second year was €252. Find the value of X .
2. (a) Find the value of $\frac{3x - 2y - 1}{5}$ when $x = 13$ and $y = 14$.
- (b) (i) Find the value of 3^6 .
- (ii) Write 27 in the form 3^k , where $k \in \mathbf{N}$.
- (iii) Find the value of x for which $27 \times 3^x = \frac{1}{729}$.
- (c) Let $f(x) = x^3 + x^2 - 4x - 4$.
- (i) Verify that $f(-2) = 0$.
 - (ii) Solve the equation
$$x^3 + x^2 - 4x - 4 = 0.$$

3. (a) Simplify $x(2x + 7) - 3(x - 4)$.

(b) (i) Solve for x and y

$$\begin{aligned}x + y &= 7 \\x^2 + y^2 &= 29.\end{aligned}$$

(ii) Which one of the values of y in (i) above satisfies the inequality

$$6 - 2y < 0?$$

Justify your answer.

(c) A rectangle has length $2\sqrt{x}$ cm and width \sqrt{x} cm.
The length of a diagonal of the rectangle is $\sqrt{45}$ cm.

(i) Find the area of the rectangle.

(ii) The area of a square is twice the area of the rectangle.
Find the length of a side of the square.

4. (a) Given that $i^2 = -1$, simplify

$$2(3 - 5i) + 7i(2 + 3i)$$

and write your answer in the form $x + yi$, where $x, y \in \mathbf{R}$.

(b) Let $u = 3 + 5i$.

(i) Show that u is a solution of the equation $z^2 - 6z + 34 = 0$.

(ii) Express $\frac{17}{u}$ in the form $x + yi$.

(c) Let $z = 3 - 4i$.

(i) Calculate $|z|$.

(ii) Find the real numbers p and q such that

$$|z|(p + qi) + (q - pi) = 17 + 7i.$$

5. (a) The first term of a geometric sequence is 2 and the common ratio is 3. Find the second term of the sequence.
- (b) The first term of an arithmetic series is -2 and the second term is 4.
- (i) Find d , the common difference.
- (ii) Find T_{10} , the tenth term of the series.
- (iii) The k th term of the series is 292. Find k .
- (iv) Find S_{20} , the sum of the first 20 terms of the series.
- (c) The first two terms of a geometric series are $-6 + 12 + \dots$
- (i) Find r , the common ratio.
- (ii) Find T_7 , the seventh term of the series.
- (iii) Starting with the first term, how many terms of the series must be added to give a sum of 30?
6. (a) Let $g(x) = 4 - kx$.
Given that $g(-5) = 34$, find the value of k .
- (b) Let $h(x) = x(1 - x^2)$, where $x \in \mathbf{R}$.
- (i) Verify that $h(3) + h(-3) = 0$.
- (ii) Find the values of x for which $h'(x) = -11$, where $h'(x)$ is the derivative of $h(x)$.
- (c) Let $f(x) = x^3 - 6x^2 + 9x - 3$, where $x \in \mathbf{R}$.
- (i) Find the co-ordinates of the local maximum point and of the local minimum point of the curve $y = f(x)$.
- (ii) Draw the graph of the function f in the domain $0 \leq x \leq 4$.
- (iii) Use your graph to estimate the range of values of x for which $x < 3$ and $f(x) \geq 0$.

7. (a) Differentiate $3x^5 - 7x^2 + 9x$ with respect to x .

(b) (i) Given that $y = (x^2 - 4x)^5$, find the value of $\frac{dy}{dx}$ when $x = 2$.

(ii) Differentiate $\frac{x^2 - 1}{x^2 + 1}$ with respect to x .

Write your answer in the form $\frac{kx}{(x^2 + 1)^n}$, where $k, n \in \mathbf{N}$.

(c) A ball is fired straight up in the air.

The height, h metres, of the ball above the ground is given by

$$h = 30t - 5t^2$$

where t is the time in seconds after the ball was fired.

(i) After how many seconds does the ball hit the ground?

(ii) Find the speed of the ball after 2 seconds.

(iii) Find the maximum height reached by the ball.

8. (a) Let $g(x) = 2(6 - 3x)$, where $x \in \mathbf{R}$.

Find the value of x for which $g(x) = 0$.

(b) Differentiate $2x^2 - 5x$ with respect to x from first principles.

(c) Let $f(x) = \frac{1}{x+1}$, $x \in \mathbf{R}$, $x \neq -1$.

(i) Find $f'(x)$, the derivative of $f(x)$.

(ii) Find the two values of x at which the slope of the tangent to the curve $y = f(x)$ is -1 .

(iii) One of these tangents intersects the positive y -axis.
Find the equation of this tangent.

Blank Page

Blank Page

Blank Page