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LEAVING CERTIFICATE EXAMINATION, 2001

MATHEMATICS — ORDINARY LEVEL

PAPER 2 (300 marks)

MONDAY, 11 JUNE — MORNING, 9.30 to 12.00

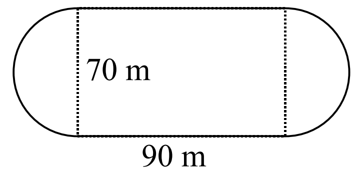
Attempt **FIVE** questions from **Section A** and **ONE** question from **Section B**.
Each question carries 50 marks.

WARNING: Marks may be lost if all necessary work is not clearly shown.

SECTION A

Attempt FIVE questions from this section.

1. **(a)** A running track is made up of two straight parts and two semicircular parts as shown in the diagram.

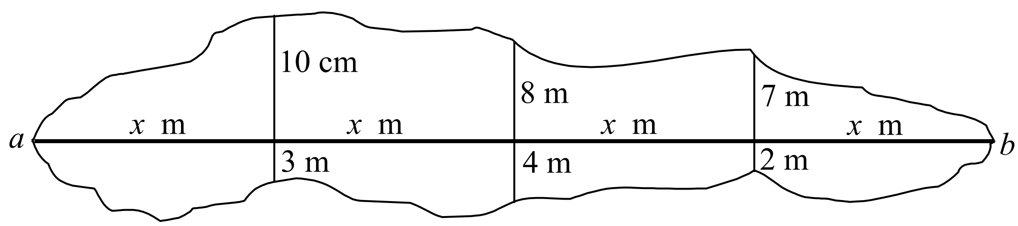


The length of each of the straight parts is 90 metres.

The diameter of each of the semicircular parts is 70 metres.

Calculate the length of the track correct to the nearest metre.

(b) The sketch shows a flood caused by a leaking underground pipe that runs from a to b.



At equal intervals of x m along $[ab]$ perpendicular measurements are made to the edges of the flood. The measurements to the top edge are 10 m, 8 m and 7 m. The measurements to the bottom edge are 3 m, 4 m and 2 m. At a and b the measurements are 0 m.

Using Simpson's Rule the area of the flood is estimated to be 672 m^2.

Find x and hence, write down the length of the pipe.

(c) Sweets, made from a chocolate mixture, are in the shape of solid spherical balls. The diameter of each sweet is 3 cm.

36 sweets fit exactly in a rectangular box which has internal height 3 cm.

(i) The base of the box is a square. How many sweets are there in each row?

(ii) What is the internal volume of the box?

(iii) The 36 sweets weigh 675 grammes.
What is the weight of 1 cm^3 of the chocolate mixture? Give your answer correct to one decimal place.

2. **(a)** The point $(t, 2t)$ lies on the line $3x + 2y + 7 = 0$.
Find the value of t.

(b) $a(4,2)$, $b(-2,0)$ and $c(0,4)$ are three points.

(i) Prove that $ac \perp bc$.

(ii) Prove that $|ac| = |bc|$.

(iii) Calculate the area of the triangle *bac*.

(iv) The diagonals of the square *bahg* intersect at *c*.
Find the co-ordinates of *h* and the co-ordinates of *g*.

(v) Find the equation of the line *bc* and show that *h* lies on this line.

3. **(a)** The circle *S* has equation $(x-3)^2 + (y-4)^2 = 25$.

(i) Write down the centre and the radius of *S*.

(ii) The point $(k, 0)$ lies on S. Find the two real values of *k*.

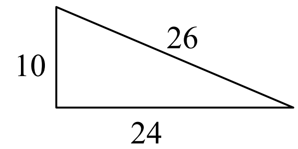
(b) Prove that the line $x - 3y = 10$ is a tangent to the circle with equation $x^2 + y^2 = 10$ and find the co-ordinates of the point of contact.

(c) *C* is a circle with centre $(0,0)$. It passes through the point $(1, -5)$.

(i) Write down the equation of *C*.

(ii) The point (p, p) lies inside *C* where $p \in \mathbf{Z}$.
Find all the possible values of *p*.

4. **(a)** Prove that the triangle with sides of lengths 10 units, 24 units and 26 units is right-angled.



(b) Prove that a line which is parallel to one side-line of a triangle, and cuts a second side, will cut the third side in the same proportion as the second.

(c) **(i)** Draw a square with sides 7 cm and mark o, the point of intersection of the diagonals.

(ii) Draw the image of the square under the enlargement with centre o and scale factor $\frac{1}{2}$.

(iii) Calculate the area of the image square.

(iv) Under another enlargement the area of the image of the square with sides 7 cm is 196 cm^2.

What is the scale factor of this englargement?

5. **(a)** $\text{Sin}\theta = \dfrac{3}{5}$ where $0° < \theta < 90°$.

Find, without using the Tables or a calculator, the value of

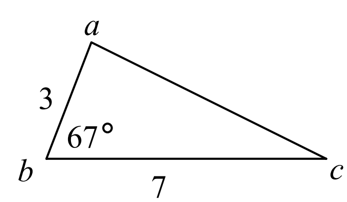
(i) $\cos\theta$

(ii) $\cos 2\theta$. [Note: $\cos 2\theta = \cos^2\theta - \sin^2\theta$.]

(b) In the triangle abc, $|ab| = 3$ units, $|bc| = 7$ units and $|\angle abc| = 67°$.

(i) Calculate the area of the triangle abc, correct to one decimal place.

(ii) Calculate $|ac|$, correct to the nearest whole number.

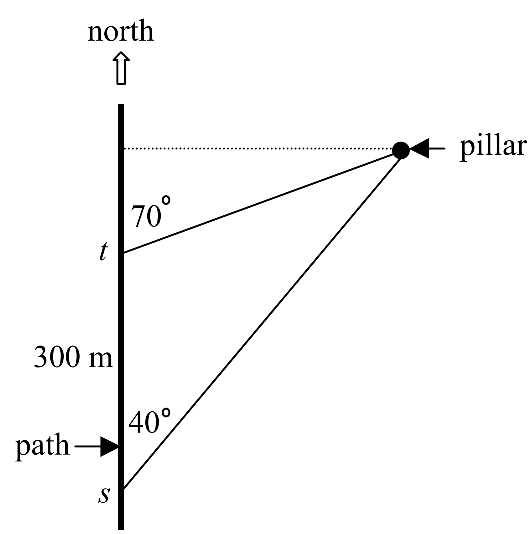


(c) s and t are two points 300 m apart on a straight path due north.

From s the bearing of a pillar is N 40° E.
From t the bearing of the pillar is N 70° E.

(i) Show that the distance from t to the pillar is 386 m, correct to the nearest metre.

(ii) Find the shortest distance from the path to the pillar, correct to the nearest metre.



6. **(a)** Sarah and Jim celebrate their birthdays in a particular week (Monday to Sunday inclusive).

Assuming that the birthdays are equally likely to fall on any day of the week, what is the probability that
(i) Sarah's birthday is on Friday
(ii) Sarah's birthday and Jim's birthday are both on Friday?

(b) **(i)** How many different arrangements can be made using all the letters of the word IRELAND?

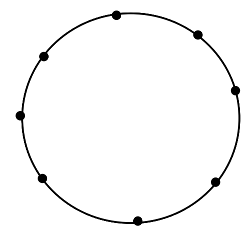
(ii) How many arrangements begin with the letter I?

(iii) How many arrangements end with the word LAND?

(iv) How many begin with I and end with LAND?

(c) **(i)** Eight points lie on a circle, as in the diagram.

How many different lines can be drawn by joining any two of the eight points?



(ii) Find the value of the natural number n such that

$$\binom{n}{2} = 105. \qquad \text{[Note: } \binom{n}{2} \text{ may also be written as } {}^{n}C_2. \text{]}$$

7. **(a)** **(i)** Calculate the mean of the following numbers

$$2,\ 3,\ 5,\ 7,\ 8.$$

(ii) Hence, calculate the standard deviation of the numbers correct to one decimal place.

(b) The following table shows the distribution of the amounts spent by 40 customers in a shop:

Amount Spent	IR£0 - IR£8	IR£8 - IR£12	IR£12 - IR£16	IR£16 - IR£20	IR£20 - IR£32
Number of Customers	2	9	13	10	6

[Note: IR£8 - IR£12 means IR£8 or over but less than IR£12 etc.]

(i) Taking mid-interval values, estimate the mean amount spent by the customers.

(ii) Copy and complete the following cumulative frequency table:

Amount Spent	< IR£8	< IR£12	< IR£16	< IR£20	< IR£32
Number of Customers					

(iii) Draw a cumulative frequency curve (ogive).

(iv) Use your curve to estimate the number of customers who spent IR£25 or more.

SECTION B

Attempt ONE question from this section.

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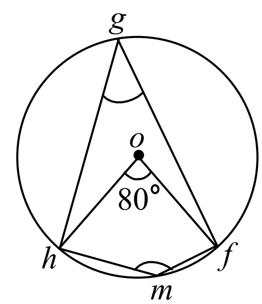
8. **(a)** The points f, g, h and m lie on a circle with centre o.



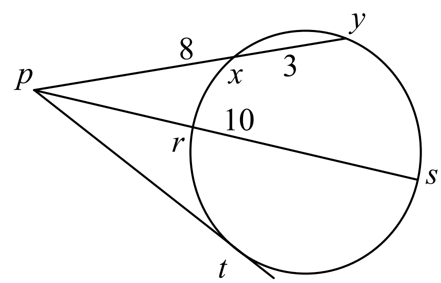
Given that $|\angle foh| = 80°$, find

(i) $|\angle fgh|$

(ii) $|\angle fmh|$.

(b) Prove that if $[ab]$ and $[cd]$ are chords of a circle and the lines ab and cd meet at the point k which is inside the circle, then $|ak|.|kb| = |ck|.|kd|$.

(c) $[xy]$ and $[rs]$ are chords of a circle which intersect at a point p outside the circle.
pt is a tangent to the circle at the point t.



Given that $|py| = 8$, $|xy| = 3$ and $|ps| = 10$,

(i) write down $|px|$

(ii) calculate $|rs|$

(iii) calculate $|pt|$, giving your answer in its simplest surd form.

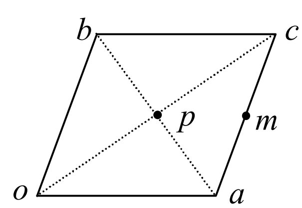
9. **(a)** Given that $\vec{p} = 5\vec{i} - 12\vec{j}$,

(i) calculate $|\vec{p}|$

(ii) write down $\vec{p}^{\perp}$ in terms of $\vec{i}$ and $\vec{j}$.

(b) **(i)** Find the scalars k and t such that $2(3\vec{i} - t\vec{j}) + k(-\vec{i} + 2\vec{j}) = t\vec{i} - 8\vec{j}$.

(ii) $oacb$ is a parallelogram where o is the origin.
p is the point of intersection of the diagonals.
m is the midpoint of $[ac]$.



Express $\vec{p}$ and $\vec{m}$ in terms of $\vec{a}$ and $\vec{b}$.

(c) Let $\vec{x} = \vec{i} + 2\vec{j}$ and $\vec{y} = 6\vec{i} + 2\vec{j}$.

(i) Calculate $\vec{x}.\vec{y}$.

(ii) Hence, find the measure of the angle between $\vec{x}$ and $\vec{y}$.

10. **(a)** Expand $(1+x)^3$ fully.
Expand $(1-x)^3$ fully.

Hence, find the real numbers a and k such that
$$(1+x)^3+(1-x)^3=a+kx^2.$$

(b) The nth term of a geometric series is given by $T_n=27\left(\frac{2}{3}\right)^n$.

(i) Write out the first three terms of the series.

(ii) Find an expression for the sum of the first five terms.

(iii) Find the sum to infinity of the series.

(c) IR£100 was invested at the beginning of each year for twenty consecutive years at 4% per annum compound interest.

Calculate the total value of the investment at the end of the twenty years, correct to the nearest IR£.

11. **(a)** Using graph paper, illustrate the set of points (x,y) that simultaneously satisfy the three inequalities:
$$y\geq 2$$
$$x+2y\leq 8$$
$$5x+y\geq -5.$$

(b) Houses are to be built on 9 hectares of land.
Two types of houses, bungalows and semi-detached houses, are possible.

Each bungalow occupies one fifth of a hectare.
Each semi-detached house occupies one tenth of a hectare.

The cost of building a bungalow is IR£ 80 000.
The cost of building a semi-detached house is IR£50 000.
The total cost of building the houses cannot be greater than IR£4 million.

(i) Taking x to represent the number of bungalows and y to represent the number of semi-detached houses, write down two inequalities in x and y and illustrate these on graph paper.

(ii) The profit on each bungalow is IR£10 000. The profit on each semi-detached house is IR£7000. How many of each type of house should be built so as to maximise profit?