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LEAVING CERTIFICATE EXAMINATION, 2001

MATHEMATICS — ORDINARY LEVEL

PAPER 1 (300 marks)

THURSDAY, 7 JUNE — MORNING, 9.30 to 12.00

Attempt **SIX QUESTIONS** (50 marks each).

WARNING: Marks may be lost if all necessary work is not clearly shown.

1. (a) A cookery book gives the following instruction for calculating the amount of time for which a turkey should be cooked:

“Allow 15 minutes per 450 grammes plus an extra 15 minutes.”

For how many hours and minutes should a turkey weighing 9 kilogrammes be cooked?

- (b) (i) The answer to $3.58 + 2.47$ was given as 6.50.

What was the percentage error correct to one decimal place?

- (ii) Calculate the value of

$$\frac{3.1 \times 10^5 - 1.5 \times 10^4}{5.9 \times 10^6}$$

and write your answer as a decimal number.

- (c) IR£5000 was invested for 3 years at compound interest.
The rate for the first year was 4%. The rate for the second year was $4\frac{1}{2}\%$.

- (i) Find the amount of the investment at the end of the second year.

At the beginning of the third year a further IR£4000 was invested.

The rate for the third year was $r\%$.

The total investment at the end of the third year was IR£9811.36.

- (ii) Calculate the value of r .

2. (a) Find the solution set of

$$11 - 2n > 3, \quad n \in \mathbf{N}.$$

- (b) Solve for x and y

$$x + 2y = 3$$

$$x^2 - y^2 = 24.$$

- (c) Solve each of the following equations for p

(i) $9^p = \frac{1}{\sqrt{3}}$

(ii) $2^{3p-7} = 2^6 - 2^5$.

3. (a) Given that $u^2 + 2as = v^2$,
calculate the value of a when $u = 10$, $s = 30$ and $v = 20$.

- (b) (i) Simplify $(x + \sqrt{x})(x - \sqrt{x})$ when $x > 0$.
(ii) Hence, or otherwise, find the value of x for which

$$(x + \sqrt{x})(x - \sqrt{x}) = 6.$$

- (c) Let $f(x) = x^3 + ax^2 + bx - 6$ where a and b are real numbers.
Given that $x - 1$ and $x - 2$ are factors of $f(x)$
(i) find the value of a and the value of b
(ii) hence, find the values of x for which $f(x) = 0$.

4. (a) Let $w = 3 - 2i$ where $i^2 = -1$.

Plot

- (i) w
(ii) iw

on an Argand diagram.

- (b) Solve

$$(x + 2yi)(1 - i) = 7 + 5i$$

for real x and for real y .

- (c) Let $z_1 = 3 + 4i$ and $z_2 = 12 - 5i$.

\bar{z}_1 and \bar{z}_2 are the complex conjugates of z_1 and z_2 , respectively.

- (i) Show that $z_1\bar{z}_2 + \bar{z}_1z_2$ is a real number.
(ii) Investigate if $|z_1| + |z_2| = |z_1 + z_2|$.

5. (a) 5, 13, 21, 29, ... is an arithmetic sequence.

Which term of the sequence is 813?

- (b) The n th term of a geometric series is given by

$$T_n = 3^n.$$

- (i) What is the value of a , the first term?
(ii) What is the value of r , the common ratio?
(iii) Show that S_{10} , the sum of the first 10 terms, is $\frac{3}{2}(3^{10} - 1)$.

- (c) The sum of the first n terms of an arithmetic series is given by

$$S_n = 4n^2 - 8n.$$

- (i) Use S_1 and S_2 to find the first term and the common difference.
(ii) Starting with the first term, how many terms of the series must be added to give a sum of 252?

6. (a) Let $g(x) = \frac{1}{x^2 + 1}$ for $x \in \mathbf{R}$.

Evaluate

- (i) $g(2)$
(ii) $g(3)$ and write your answers as decimals.

- (b) Let $f(x) = 2 - 9x + 6x^2 - x^3$ for $x \in \mathbf{R}$.

- (i) Find $f(-1)$, $f(2)$ and $f(5)$.
(ii) Find $f'(x)$, the derivative of $f(x)$.
(iii) Find the co-ordinates of the local maximum and the local minimum of $f(x)$.
(iv) Draw the graph of $f(x)$ in the domain $-1 \leq x \leq 5$.
(v) Use your graph to find the range of real values of k for which $f(x) = k$ has more than one solution.

7. (a) Differentiate with respect to x
- (i) $6x^5 + x^2$
 - (ii) $(x - 3)(x + 3)$.
- (b) (i) Find $\frac{dy}{dx}$ when $y = \frac{x^2}{x - 4}$, $x \neq 4$.
- (ii) Find the value of $\frac{dy}{dx}$ at $x = 0$ when $y = (x^2 - 7x + 1)^5$.
- (c) Two fireworks were fired straight up in the air at $t = 0$ seconds. The height, h metres, which each firework reached above the ground t seconds after it was fired is given by

$$h = 80t - 5t^2.$$

The first firework exploded 5 seconds after it was fired.

- (i) At what height was the first firework when it exploded?
- (ii) At what speed was the first firework travelling when it exploded?

The second firework failed to explode and it fell back to the ground.

- (iii) After how many seconds did the second firework reach its maximum height?

8. (a) Let $g(x) = x^4 - 32x$ for $x \in \mathbf{R}$.
- (i) Write down $g'(x)$, the derivative of $g(x)$.
 - (ii) For what value of x is $g'(x) = 0$?
- (b) Differentiate $3x^2 - x$ from first principles with respect to x .
- (c) Let $f(x) = \frac{1}{x+1}$ for $x \in \mathbf{R}$ and $x > -1$.
- (i) Find $f'(x)$.
 - (ii) Find the co-ordinates of the point on the curve of $f(x)$ at which the tangent has slope of $-\frac{1}{4}$.
 - (iii) Find the equation of the tangent to the curve which has slope of $-\frac{1}{4}$.