AN ROINN OIDEACHAIS AGUS EOLAÍOCHTA

LEAVING CERTIFICATE EXAMINATION, 2000

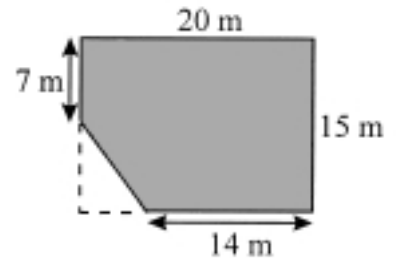# MATHEMATICS – ORDINARY LEVEL – PAPER 2
(300 marks)

FRIDAY, 9 JUNE — MORNING, 9.30 to 12.00

Attempt 5 **Questions** from Section **A** and **ONE Question** from Section **B**.
Each question carries 50 marks.

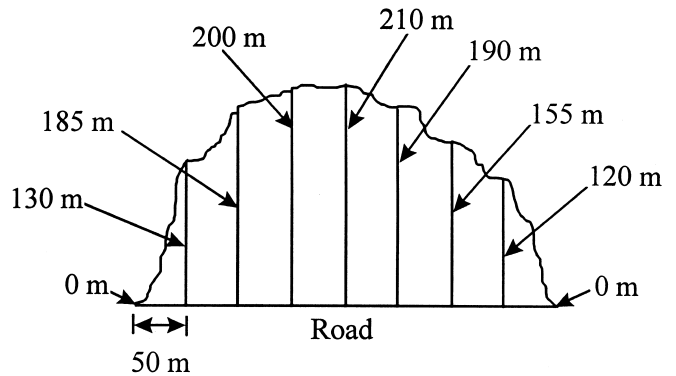**Marks may be lost if necessary work is not clearly shown.**

1. **(a)** Calculate the area of the shaded region in the diagram.

(b) The sketch shows a piece of land covered by forest which lies on one side of a straight road.

At equal intervals of 50 m along the road, perpendicular measurements of 130 m, 185 m, 200 m, 210 m, 190 m, 155 m and 120 m are made to the forest boundary.

Use Simpson's Rule to estimate the area of land covered by the forest.

[See Tables, page 42.]

Give your answer in hectares. [Note: 1 hectare = 10 000 m^2.]

(c) A candle is in the shape of a cylinder surmounted by a cone, as in the diagram.

   (i) The cone has height 24 cm and the length of the radius of its base is 10 cm.
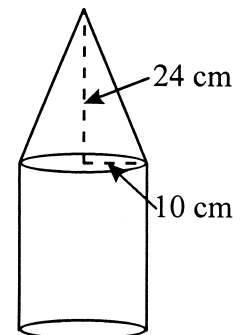
     Find the volume of the cone in terms of π.

  (ii) The height of the cylinder is equal to the slant height of the cone.

     Find the volume of the cylinder in terms of π.

 (iii) A solid spherical ball of wax with radius of length r cm was used to make the candle.

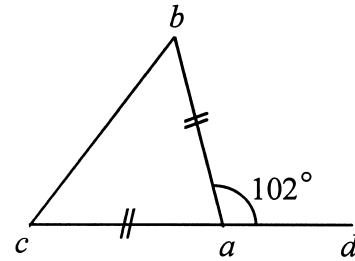   Calculate r, correct to one decimal place.

2. **(a)** Find the coordinates of the midpoint of the line segment which joins the points $(2, -3)$ and $(-8, -6)$.

(b) $a(-2, -1)$, $b(1, 0)$ and $c(-5, 2)$ are three points.

  (i) Show that $|ab| = \sqrt{10}$.

  (ii) Find $|bc|$.

  (iii) Hence, find the ratio $|ab| : |bc|$.
  Give your answer in the form $m : n$ where m and n are whole numbers.

(c) **(i)** The line L has equation $3x - 4y + 20 = 0$.
  K is the line through $p(0, 5)$ which is perpendicular to L.
  Find the equation of K.

  (ii) L cuts the x-axis at the point t.
  K cuts the x-axis at the point r.
  Calculate the area of the triangle ptr. Give your answer as a fraction.

3. **(a)** The circle C has equation $x^2 + y^2 = 16$.

  (i) Write down the length of the radius of C.

  (ii) Show, by calculation, that the point $(3, 1)$ is inside the circle.

(b) **(i)** Find the slope of the tangent to the circle $x^2 + y^2 = 29$ at the point $(2, 5)$.

  (ii) Hence, find the equation of the tangent.

(c) **(i)** The end points of a diameter of a circle are $(-2, -3)$ and $(-4, 3)$.
  Find the equation of the circle.

  (ii) The circle cuts the y-axis at the points a and b. Find $|ab|$.

  (iii) c and d are points on the circle such that $abcd$ is a rectangle.
  Find the area of the rectangle $abcd$.

4. **(a)** In the diagram, $|ab| = |ac|$ and $|\angle bad| = 102°$.

   (i) Find $|\angle cab|$.

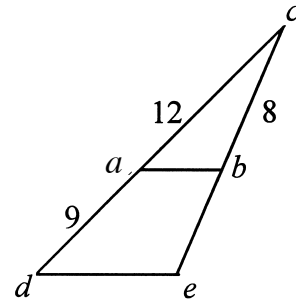   **(ii)** Find $|\angle abc|$.

(b) Prove that in a right-angled triangle, the square of the length of the side opposite to the right-angle is equal to the sum of the squares of the lengths of the other two sides.

(c) The triangle *cde* is the image of the triangle *cab* under an enlargement with centre *c*. $|ca| = 12, |ad| = 9$ and $|cb| = 8$.

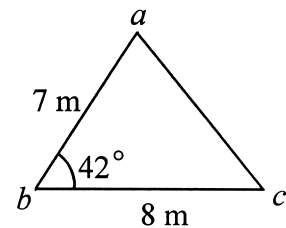   **(i)** Find the scale factor of the enlargement.

   (ii) Find $|be|$.

   (iii) The area of the triangle *cde* is 98 square units. Find the area of the triangle *cab*.

5. **(a)** In the triangle abc, $|ab| = 7$ m, $|bc| = 8$ m and $|\angle abc| = 42°$.

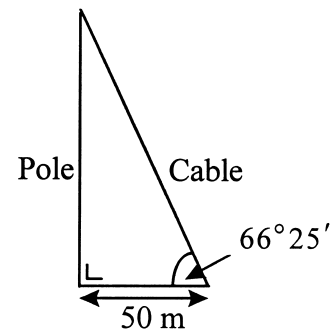   Calculate the area of the triangle, correct to one place of decimals.

(b) The diagram shows a vertical pole which stands on level ground.

   A cable joins the top of the pole to a point on the ground which is 50 m from the base of the pole.

   The cable makes an angle of 66°25′ with the ground.

   (i) Find the height of the pole, correct to the nearest metre.

   (ii) Find the length of the cable, correct to the nearest metre.
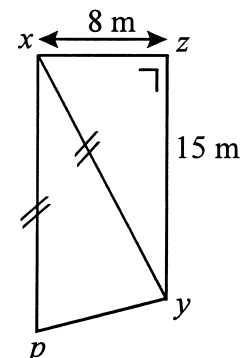
(c) **(i)** In the diagram, the triangle *zxy* is right-angled. $|zx| = 8$ m and $|zy| = 15$ m. Find $|xy|$.

   (ii) *xp* is parallel to *zy*. $|xp| = |xy|$, as shown. Calculate $|py|$, correct to the nearest metre.

6. (a) To go to work, a woman can walk or travel by bus or travel by car with a neighbour. To return home, she can walk or travel by bus.

    (i) In how many different ways can the woman go to and return from work on any one day?

    (ii) List all of these different ways.

(b) In a class, there are 15 boys and 13 girls. Four boys wear glasses and three girls wear glasses.

    A pupil is picked at random from the class.

    (i) What is the probability that the pupil is a boy?

    (ii) What is the probability that the pupil wears glasses?

    (iii) What is the probability that the pupil is a boy who wears glasses?

    A girl is picked at random from the class.

    (iv) What is the probability that she wears glasses?

(c) **(i)** How many different five-digit numbers can be formed from the digits 2, 3, 4, 5, 6? Each digit can be used once only in each number.

    (ii) How many of the numbers are even?

    (iii) How many of the numbers are less than 40 000?

    (iv) How many of the numbers are both even and less than 40 000?

7. (a) Find the weighted mean of 11, 15, 19 and 21 if the weights are 2, 3, 1 and 2 respectively.

(b) The table shows the distribution of points obtained by 50 people who took a driving test.

Points obtained	0 – 20	20 – 40	40 – 80	80 – 100
Number of people	4	8	28	10

    (i) Draw a histogram to illustrate the data.

    (ii) To pass the driving test a person must obtain 65 points or more. What is the greatest possible number of people who passed the test?

(c) The table below refers to the number of emergency calls recorded at a fire station each week for 52 weeks.

Number of emergency calls	0 – 10	11 – 20	21 – 30	31 – 40	41 – 50	51 – 60	61 – 70
Number of weeks	6	8	11	12	7	5	3

    (i) Copy and complete the following cumulative frequency table:

Number of emergency calls	≤ 10	≤ 20	≤ 30	≤ 40	≤ 50	≤ 60	≤ 70
Number of weeks	6						52

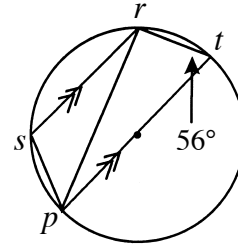   **(ii)** Draw the cumulative frequency curve.

    (iii) Use your graph to estimate the interquartile range.

    (iv) Use your graph to estimate the number of weeks during which more than 56 emergency calls were recorded.

8. **(a)** In the diagram, [pt] is a diameter of the circle.
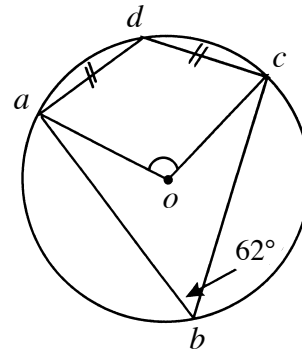sr is parallel to pt and $|\angle ptr| = 56°$.

    (i) Write down the value of $|\angle prt|$.

    (ii) Find the value of $|\angle prs|$.

(b) Prove that the degree-measure of an angle subtended at the centre of a circle by a chord is equal to twice the degree-measure of any angle subtended by the chord at a point of the arc of the circle which is on the same side of the chordal line as is the centre.

(c) In the diagram, o is the centre of the circle.

a, b, c and d are points on the circle.

$|da| = |dc|$ and $|\angle abc| = 62°$.

    (i) Find $|\angle aoc|$, where $\angle aoc$ is obtuse.

    (ii) Find $|\angle adc|$.

    (iii) Find $|\angle oad|$.

9. **(a)** Let $\vec{x} = \vec{i} + \vec{j}$ and $\vec{y} = 2\vec{i} + 5\vec{j}$.
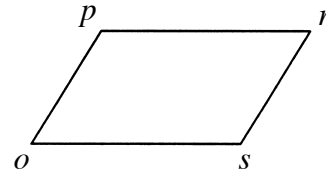
Express, in terms of $\vec{i}$ and $\vec{j}$,

    (i) $3\vec{x} + \vec{y}$

    (ii) $\overrightarrow{xy}$.

(b) osrp is a parallelogram where o is the origin.

    (i) Copy the diagram and show on it $\vec{k}$ and $\vec{m}$ such that
$\vec{k} = \vec{s} + 2\vec{p}$ and $\vec{m} = 2\vec{s} + \vec{p}$.

    (ii) Express $\vec{k} + \vec{m}$ in terms of $\vec{r}$.

(c) $\vec{a} = 5\vec{i} + 12\vec{j}$ and $\vec{b} = 3\vec{i} - 4\vec{j}$.

    (i) Write down $\vec{a}^{\perp}$ and $\vec{b}^{\perp}$ in terms of $\vec{i}$ and $\vec{j}$.

    (ii) Evaluate $|\vec{a}^{\perp}|$ and $|\vec{b}^{\perp}|$.

    (iii) Find the scalar k such that $|\vec{a}^{\perp} + \vec{b}^{\perp}| = k(|\vec{a}^{\perp}| - |\vec{b}^{\perp}|)$.
Give your answer in the form $\sqrt{n}$, where $n \in \mathbf{N}$.

10. (a) Expand $(1 + x)^3$ in ascending powers of x.

Show that $(1 + \sqrt{3})^3 = 10 + 6\sqrt{3}$.

(b) **(i)** Find the sum to infinity of the geometric series

$$\frac{4}{5} + \frac{4}{50} + \frac{4}{500} + \cdots$$

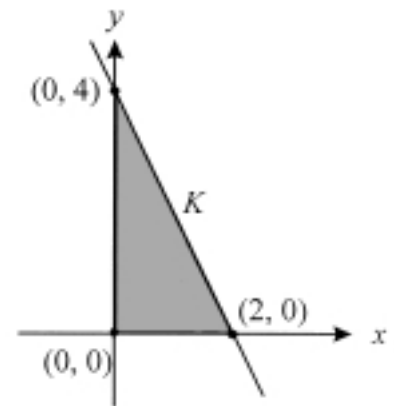**(ii)** Hence, show that $1.\dot{8} = \frac{17}{9}$.

(c) A person invests IR£1000 at the beginning of each year for 3 consecutive years at 8% per annum compound interest. Tax at 24% is deducted at the end of each year from the interest earned.

Find

(i) the value of the first investment at the end of the third year, correct to the nearest penny

(ii) the total value of all the investments at the end of the third year, correct to the nearest penny.

11. (a) The line K passes through the points $(2, 0)$ and $(0, 4)$.

(i) Find the equation of the line K.

(ii) Write down three inequalities which define the shaded region in the diagram.

(b) Two types of machines, type A and type B, can be purchased for a new factory. Each machine of type A costs IR£1600. Each machine of type B costs IR£800. The purchase of the machines can cost, at most, IR£27 200.

Each machine of type A needs 90 m^2 of floor space in the factory.

Each machine of type B needs 54 m^2 of floor space.

The maximum amount of floor space available for the machines is 1620 m^2.

(i) If x represents the number of machines of type A and y represents the number of machines of type B, write down two inequalities in x and y and illustrate these on graph paper.

(ii) The daily income from the use of each machine of type A is IR£75. The daily income from the use of each machine of type B machine is IR£42. How many of each type of machine should be purchased so as to maximise daily income?

(iii) What is the maximum daily income?