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LEAVING CERTIFICATE EXAMINATION, 1999

MATHEMATICS — ORDINARY LEVEL— PAPER 2 (300 marks)

FRIDAY, 11 JUNE — MORNING, 9.30 to 12.00

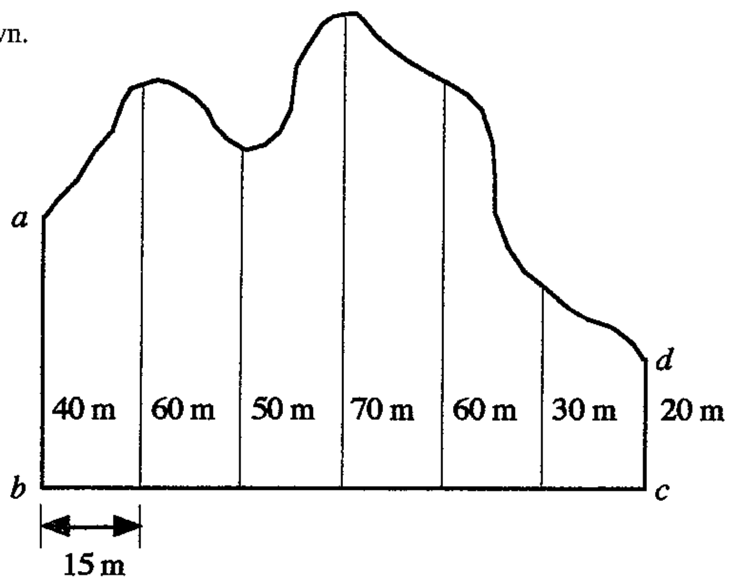
Attempt **5 Questions** from Section **A** and **ONE Question** from Section **B.**
Each question carries 50 marks.

Marks may be lost if necessary work is not clearly shown or if you do not indicate where a calculator has been used.

SECTION A

1. (a) The area of a square is 36 cm^2.
Find the length of a side of the square.

(b) A sketch of a piece of land *abcd* is shown.



At equal intervals of 15 m along [*bc*], perpendicular measurements of 40 m, 60 m, 50 m, 70 m, 60 m, 30 m and 20 m are made to the top boundary.

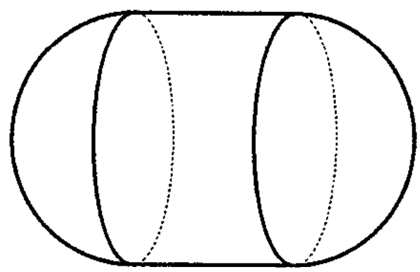
Use Simpson's Rule to estimate the area of the piece of land. [See Tables, page 42].

(c) **(i)** Write down, in terms of π and r, the volume of a hemisphere with radius of length r.

(ii) A fuel storage tank is in the shape of a cylinder with a hemisphere at each end, as shown.

The capacity (internal volume) of the tank is 81π m^3.
The ratio of the capacity of the cylindrical section to the sum of the capacities of the hemispherical ends is 5:4.

Calculate the internal radius length of the tank.



2. **(a)** The point $(k, 1)$ lies on the line $4x - 3y + 15 = 0$.

Find the value of k.

(b) $p(4, 3)$, $q(-1, 0)$ and $r(10, 3)$ are three points.

(i) Find the slope of pq.

(ii) Find the equation of the line through r which is parallel to pq.

(iii) Find the equation of the line which is perpendicular to pq and which contains the origin.

(c) $a(0, 5)$, $b(x, 10)$ and $c(2x, x)$ are three points.

Find $|ab|$ in terms of x.

If $|ab| = |bc|$, calculate the two possible values of x.

3. **(a)** C is a circle with centre $(0, 0)$ passing through the point $(8, 6)$.

Find

(i) the radius length of C

(ii) the equation of C.

(b) The points $(-1, -1)$ and $(3, -3)$ are the end points of a diameter of a circle S.

(i) Find the coordinates of the centre of S.

(ii) Find the radius length of S.

(iii) Find the equation of S.

(c) A circle K has equation

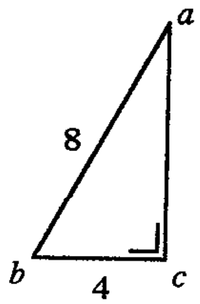
$$x^2 + y^2 = 13.$$

T is a tangent to K at $(-2, -3)$.
Find the equation of T.

Find the equation of the other tangent to K which is parallel to T.

4. **(a)** abc is a triangle with $|ab| = 8$, $|bc| = 4$ and $|\angle acb| = 90°$.

Calculate $|ac|$, correct to two places of decimals.



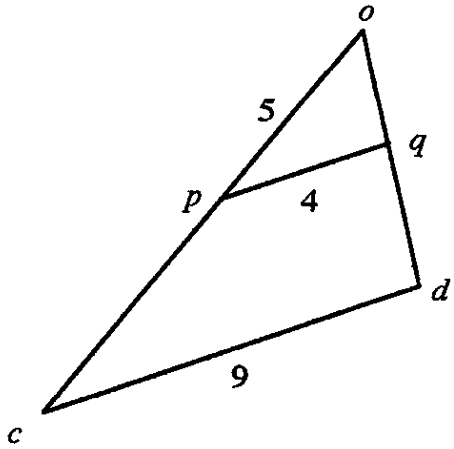
(b) Prove that the sum of the lengths of any two sides of a triangle is greater than that of the third side.

(c) The triangle ocd is the image of the triangle opq under the enlargement, centre o, with $|pq| = 4$, $|op| = 5$ and $|cd| = 9$.

(i) Find the scale factor of the enlargement.

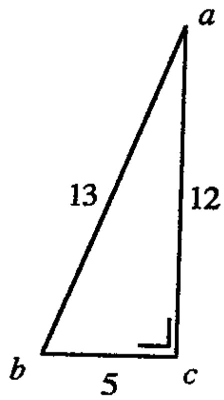
(ii) Find $|pc|$.

(iii) The area of the triangle ocd is 60.75 square units. Find the area of the triangle opq.



5. **(a)** abc is a right-angled triangle with $|\angle acb| = 90°$, $|ab| = 13$, $|bc| = 5$ and $|ac| = 12$.

Find, as fractions, the value of $\sin \angle abc$ and the value of $\tan \angle bac$.

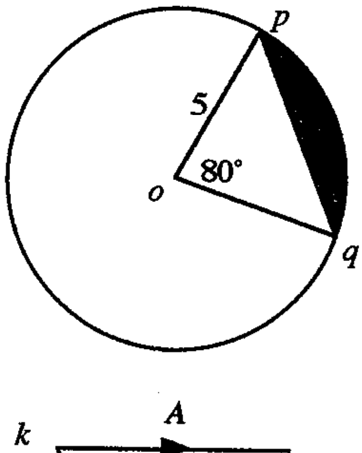


(b) In the diagram, o is the centre of the circle with radius length 5 and p and q are points on the circle. $|\angle poq| = 80°$.

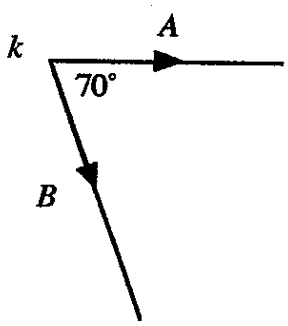
Find, correct to two places of decimals,

(i) the area of triangle poq

(ii) the area of the shaded region, taking $\pi = 3.14$.



(c) Two ships, A and B, leave port k at noon. A is travelling due East and B is travelling East 70° South, as shown.
Calculate, to the nearest km, the distance between A and B when A is 8 km from k and B is 12 km from k.



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6. **(a)** **(i)** In how many ways can a team of 5 players be chosen from a panel of 8 players?

(ii) If a certain player must be on the team, in how many ways can the team then be chosen?

(b) **(i)** In how many different ways can the 5 letters of the word A N G L E be arranged?

(ii) How many of these arrangements begin with a vowel?

(iii) In how many of the arrangements do the two vowels come together?

(c) Twelve blood samples are tested in a laboratory. Of these it is found that five blood samples are of type A, four of type B and the remaining three are of type O.

Two blood samples are selected at random from the twelve.

What is the probability that

(i) the two samples are of type A

(ii) one sample is of type B and the other sample is of type O

(iii) the two samples are of the same blood type?

7. **(a)** Four people have a meal in a restaurant. The average cost of the meal per person is IR£12.50, excluding the service charge.

What is the total bill for the four people if a 10% service charge is added?

(b) The cumulative frequency table below shows the distribution of ages of 110 people living in an estate.

Age in years	≤ 5	≤ 10	≤ 20	≤ 35	≤ 50	≤ 60
Number of people	5	15	40	90	105	110

(i) Draw the cumulative frequency curve, putting number of people on the vertical axis.

(ii) Use your curve to estimate the median age.

(iii) Use your curve to estimate the number of people who are more than 15 years of age.

(c) The number of minutes taken by 20 pupils to answer a short question is shown in the following distribution table:

Minutes	2-4	4-6	6-8	8-10
Number of pupils	6	9	4	1

By taking the data at mid-interval values, calculate

(i) the mean number of minutes taken per pupil

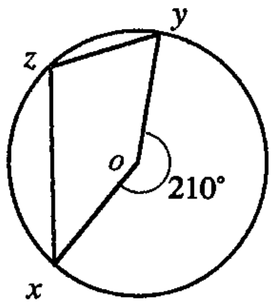
(ii) the standard deviation, correct to one place of decimals.

SECTION B

Attempt ONE question.

8. **(a)** In the diagram, o is the centre of the circle and $|\angle xoy| = 210°$.

Find $|\angle xzy|$.



(b) Prove that a line is a tangent to a circle at a point t of the circle if and only if it passes through t and is perpendicular to the line through t and the centre.

(c) The line kd is a tangent to the circle at a.

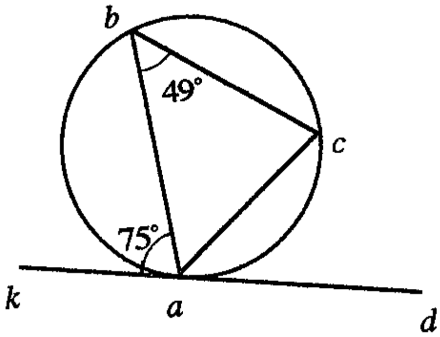
$|\angle kab| = 75°$ and $|\angle abc| = 49°$.

Find

(i) $|\angle bca|$

(ii) $|\angle bac|$

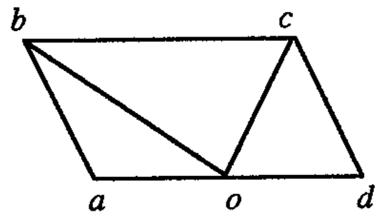
(iii) $|\angle cad|$.



9. **(a)** $abcd$ is a parallelogram. The midpoint of $[ad]$ is o, where o is the origin.

(i) Express $\vec{ab}$ in terms of $\vec{a}$ and $\vec{b}$.

(ii) Show that $\vec{c} = \vec{b} - 2\vec{a}$.



(b) Let $\vec{p} = 6\vec{i} - 2\vec{j}$ and $\vec{q} = 2\vec{i} + 3\vec{j}$.

(i) Express $\vec{q} - \vec{p}$ in terms of $\vec{i}$ and $\vec{j}$.

(ii) Calculate $|\vec{q} - \vec{p}|$.

(iii) Calculate $\vec{p}.\vec{q}$.

(c) Let $\vec{x} = 8\vec{i} - 2\vec{j}$ and $\vec{y} = 2\vec{i} + 4\vec{j}$.

Write $\vec{x}^{\perp}$ and $\vec{y}^{\perp}$ in terms of $\vec{i}$ and $\vec{j}$.

Find the value of the scalar m and the value of the scalar n for which

$$\vec{x}^{\perp} + m\vec{y}^{\perp} = 3\vec{i} - n\vec{j}.$$

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10. **(a)** Expand $(1 - x)^4$ in ascending powers of x.

(b) Find the sum to infinity of the geometric series

$$\frac{63}{100} + \frac{63}{10\,000} + \frac{63}{1\,000\,000} + \dots .$$

Using this series, show that

$$1.\dot{6}\dot{3} = \frac{18}{11} .$$

(c) A company invested IR£x in new equipment at the beginning of each year for three consecutive years. The equipment depreciated at the rate of 20% per annum.

Write, in terms of x, the value of the first investment of IR£x at the end of the first year.

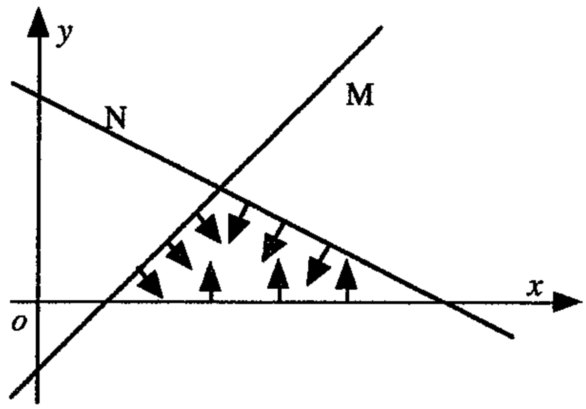
The value of the first investment of IR£x at the end of the third year is IR£10 240.

Find the value of x.

Find the total value of all the investments at the end of the third year.

11. **(a)** The equation of the line M is $x - y - 1 = 0$ and the equation of the line N is $x + 2y - 6 = 0$.

Write down the three inequalities which define the triangular region indicated in the diagram.



(b) A company uses small trucks and large trucks to transport its products in crates. The crates are all of the same size.

On a certain day 10 truck drivers at most are available. Each truck requires one driver only.

Small trucks take 10 minutes each to load and large trucks take 30 minutes each to load. The total loading time must not be more than 3 hours. Only one truck can be loaded at a time.

(i) If x represents the number of small trucks used and y represents the number of large trucks used, write down two inequalities in x and y.

Illustrate these on graph paper.

(ii) Each small truck carries 30 crates and each large truck carries 70 crates. How many of each type of truck should be used to maximize the number of crates to be transported that day?