LEAVING CERTIFICATE EXAMINATION, 1997 57895

MATHEMATICS — ORDINARY LEVEL — PAPER I (300 marks)

THURSDAY, 12 JUNE — MORNING, 9.30 to 12.00

Attempt **SIX QUESTIONS** (50 marks each)

Marks may be lost if all your work is not clearly shown or if you do not indicate where a calculator has been used.

---

1. **(a)** A machine broke down at 0935 hours. It was repaired at 1210 hours. For how many hours and minutes was the machine out of order?

(b) IR£2500 was invested for three years at compound interest.

The rate of interest was 4% per annum for the first year and 3% per annum for the second year.

Calculate the amount of the investment after two years.

If the investment amounted to IR£2744.95 after three years, calculate the rate of interest per annum for the third year.

(c) **(i)** The length and breadth of a rectangle are in the ratio 9 : 5, respectively. The length of the rectangle is 22.5 cm. Find its breadth.

(ii) Tea served in a canteen is made from a mixture of two different types of tea, type A and type B. Type A costs IR£4.05 per kg. Type B costs IR£4.30 per kg. The mixture costs IR£4.20 per kg.

If the mixture contains 7 kg of type A, how many kilograms of type B does it contain?

2. **(a)** Solve for x

$$3(2x - 1) = 4x.$$

(b) Find the solution set E of $9 - 2x \geq 7$, $x \in \mathbf{N}$.

Find the solution set H of $\frac{1}{4}x - \frac{1}{3} \leq \frac{5}{12}$, $x \in \mathbf{N}$.

Write down the elements of the set $H \setminus E$.

(c) Simplify

$$(\sqrt{x} + \frac{3}{\sqrt{x}})(\sqrt{x} - \frac{3}{\sqrt{x}}) \quad \text{where } x > 0.$$

Hence solve for x

$$(\sqrt{x} + \frac{3}{\sqrt{x}})(\sqrt{x} - \frac{3}{\sqrt{x}}) = 8 \quad \text{where } x > 0.$$

OVER →

3. **(a)** Express p in terms of q and t when

$$2p - q = 3(p - t).$$

(b) Solve the equation

$$2x^3 + 3x^2 - 5x - 6 = 0.$$

(c) Let $f(x) = (2 + x)(3 - x)$, $x \in \mathbf{R}$.
Write down the solutions (roots) of $f(x) = 0$.

Let $g(x) = 3x - k$.
The equation $f(x) + g(x) = 0$ has equal roots. Find the value of k.

4. **(a)** Simplify

$$3(1 + 5i) + i(3 - 2i)$$

and express your answer in the form $p + qi$, where $p, q \in \mathbf{R}$ and $i^2 = -1$.

(b) **(i)** For what values of a is

$$|a + 8i| = 10 \quad \text{where } a \in \mathbf{R}?$$

(ii) If $w = 4i$, verify that

$$w^3 - w^2 + 16w - 16 = 0.$$

(c) Let $z = 1 + i$ and let $\overline{z}$ be the complex conjugate of z.
Express $\frac{z}{\overline{z}}$ in the form $x + yi$, $x, y \in \mathbf{R}$.

Hence, solve

$$k\left(\frac{z}{\overline{z}}\right) + tz = -3 - 4i$$

for real k and real t.

5. (a) $T_1 + T_2 + T_3 + \ldots\ldots\ldots$ is a geometric series.
The first term, T_1, is 1 and the common ratio is 2.

Show that

$$T_3 + T_5 = 2(T_2 + T_4).$$

(b) The first four terms of an arithmetic sequence are given as

$$a, -4, b, 6, \ldots\ldots\ldots$$

Find

(i) the value of a and the value of b

(ii) T_5, the fifth term.

(c) In an arithmetic series

$$S_n = n^2 + n,$$

where S_n is the sum to the first n terms.

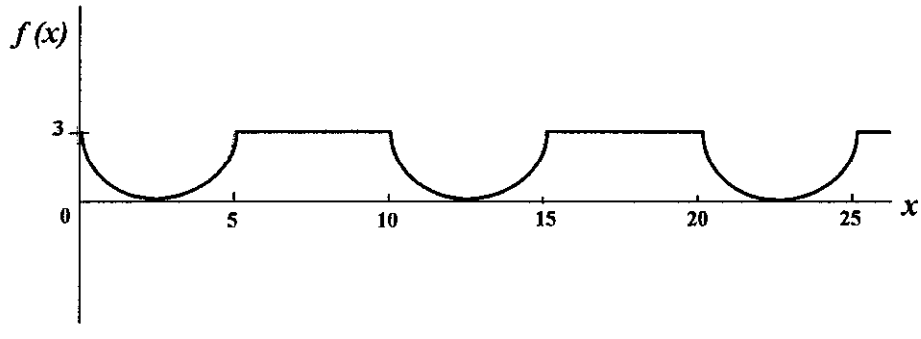
Write down

(i) S_{10}, the sum to 10 terms

(ii) S_{11}, the sum to 11 terms

(iii) T_{11}, the 11th term.

6. (a)



The graph shows portion of a periodic function $f : x \to f(x)$.

Write down the period and range of the function.

What is the value of $f(77.5)$?

(b) Differentiate from first principles

$$3x^2 - 2$$

with respect to x.

(c) Let $f(x) = ax^3 + bx + c$, for all $x \in \mathbf{R}$ and for $a, b, c \in \mathbf{R}$.

Use the information which follows to find the value of a, of b and of c:

(i) $f(0) = 3$

(ii) the slope of the tangent to the curve of $f(x)$ at $x = 1$ is -18

(iii) the curve of $f(x)$ has a local minimum at $x = 2$.

OVER →

7. **(a)** Differentiate with respect to x

(i) $-x^2$

(ii) $x^4 + x + 1$.

(b) **(i)** Find $\frac{dy}{dx}$ when $y = (x^2 - 3)(1 - x)$.

(ii) Find the value of $\frac{dy}{dx}$ at $x = -1$ when $y = (3x + 1)^4$.

(c) The distance s metres of an object from a fixed point at t seconds is given by

$$s = \frac{t + 1}{t + 3}.$$

(i) At what time is the object 0.75 m from the fixed point?

(ii) What is the speed of the object, in terms of t, at t seconds?

(iii) After how many seconds will the speed of the object be less than 0.02 m/s?

8. **(a)** Let $f(x) = x^2 - 4x$, for $x \in \mathbf{R}$.

Find $f'(x)$, the derivative of $f(x)$.

For what value of x is $f'(x) = 0$?

(b) Find the equation of the tangent to the curve

$$y = x^3 - 4x + 7$$

at the point where $x = 1$.

(c) Draw the graph of

$$g(x) = \frac{1}{x + 2}$$

for $0 \leq x \leq 4$, $x \in \mathbf{R}$.

Using the same axes and the same scales draw the graph of

$$h(x) = x - 2.$$

Show how your graphs may be used to estimate the value of $\sqrt{5}$.