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LEAVING CERTIFICATE EXAMINATION, 1996

MATHEMATICS — ORDINARY LEVEL — PAPER 2 (300 marks)

51884

FRIDAY, 7 JUNE — MORNING, 9.30 to 12.00

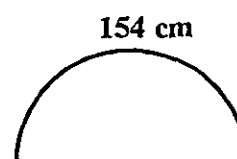
Attempt 5 Questions from Section A and ONE Question from Section B. Each question carries 50 marks.
Marks may be lost if necessary work is not shown or if you do not indicate where a calculator has been used.

SECTION A

1. (a) A piece of wire of length 154 cm is in the shape of a semicircle.

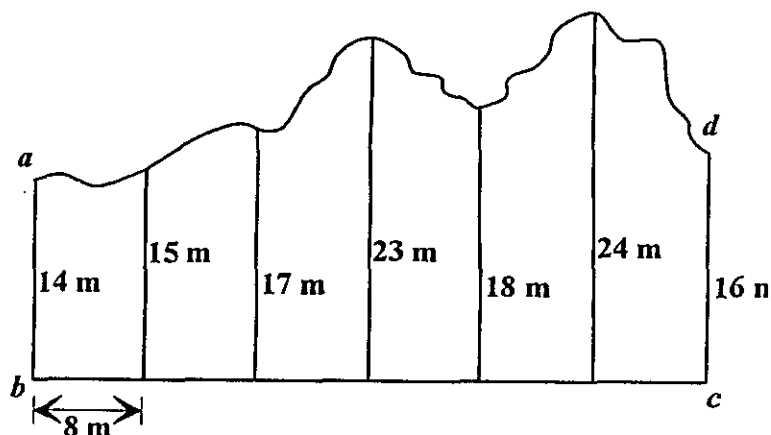
Find the radius length of the semicircle.

Take $\pi = \frac{22}{7}$.



- (b) A sketch to estimate the area of a building site $abcd$ is shown. At intervals of 8 m along $[bc]$, perpendicular measurements of 14 m, 15 m, 17 m, 23 m, 18 m, 24 m and 16 m are made to the top boundary.

Use Simpson's Rule to estimate the area of the building site. [See Tables, page 42].



- (c) A solid cylinder, made of lead, has a radius of length 15 cm and height of 135 cm.
Find its volume in terms of π .

The solid cylinder is melted down and recast to make four identical right circular solid cones. The height of each cone is equal to twice the length of its base radius.

Calculate the base radius length of the cones.

2. (a) The line L contains the points $p(3,-1)$ and $q(0,2)$.

(i) Find the slope of L .

(ii) Find the equation of L .

(iii) L intersects the x axis at the point r . Find the coordinates of r .

(iv) Calculate the ratio

$$\frac{\text{area of triangle } rgo}{\text{area of triangle } pqo},$$

where o is the origin.

(b) The equation of the line M is $y - 4x - c = 0$.

M contains the point $p(1,6)$.

(i) Find the value of c .

(ii) The origin is the midpoint of $[pq]$.

Find the equation of the line K if K is parallel to M and K contains the point q .

(iii) Find the equation of the line L if L is perpendicular to M and L contains the point q .

3. (a) The equation of a circle is

$$x^2 + y^2 = 36.$$

(i) Write down its radius length.

(ii) Verify, by calculation, that the point $(2,3)$ is inside the circle.

(b) The points $(1,0)$ and $(4,4)$ are the end points of a diameter of a circle C .

(i) Find the coordinates of the centre of C .

(ii) Find the radius length of C .

(iii) Find the equation of C .

(c) A circle K has equation

$$x^2 + y^2 = 25.$$

(i) T is a tangent to K at $(3,4)$.

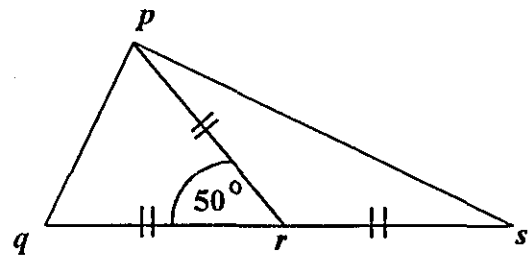
Find the equation of T .

(ii) Find the equation of the other tangent to K which is parallel to T .

4. (a) $|pr| = |qr| = |rs|$ and $|\angle prq| = 50^\circ$.

Find

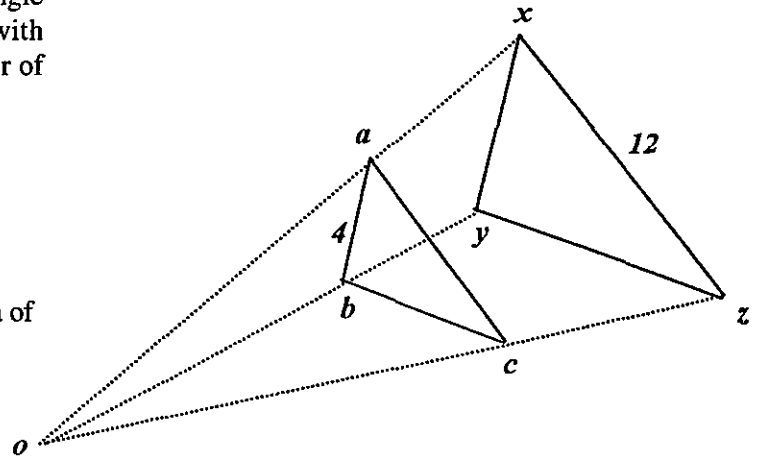
- (i) $|\angle pqr|$
 (ii) $|\angle psr|$.



- (b) Prove that a line which is parallel to one side-line of a triangle, and cuts a second side, will cut the third side in the same proportion as the second.

- (c) The triangle xyz is the image of the triangle abc under the enlargement, centre o , with $|ab| = 4$ and $|xz| = 12$. The scale factor of the enlargement is 1.5.

- (i) Find $|xy|$.
 (ii) Find $|ac|$.
 (iii) If the area of triangle abc is 12.2 square units, calculate the area of triangle xyz .



5. (a) Find the length of an arc of a circle of radius length 6 cm subtending an angle of 120° at the centre.

Give your answer in terms of π .

- (b) A and B are acute angles where $\sin A = \frac{3}{5}$ and $\cos B = \frac{5}{13}$.

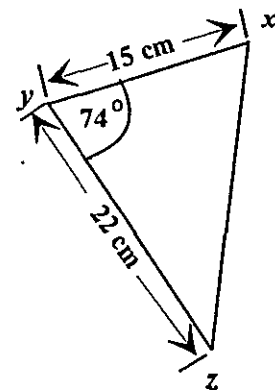
Find, as fractions, the value of $\cos A$ and the value of $\sin B$.

Find the value of $\sin(A + B)$, giving your answer as a single fraction.

- (c) xyz is a triangle where $|xy| = 15$ cm,
 $|yz| = 22$ cm and $|\angle xyz| = 74^\circ$.

Find

- (i) $|xz|$, correct to the nearest cm
 (ii) $|\angle yxz|$, correct to the nearest degree.



6. (a) A bag contains 24 beads of which 12 are red, 8 are blue and 4 are white.
A bead is taken at random from the bag.
What is the probability that the colour of the bead is
- blue
 - red or white?
- (b) There are 5 horses — *A*, *B*, *C*, *D* and *E* — in a race. Each horse takes a different time to complete the race. On completing the race,
- in how many different placing arrangements can the 5 horses finish?
 - if *A* is placed first and *B* last, in how many different placing arrangements can the other horses finish?
- (c) A committee of two people is chosen at random from 4 men and 5 women.
What is the probability that there will be one woman or two women on the committee?

7. (a) The numbers 3, 5, 6, x , 9, 2 have a mean of 6.

Find x .

- (b) The cumulative frequency table below shows the number of minutes taken by 80 people to complete a crossword:

Minutes	≤ 10	≤ 20	≤ 30	≤ 40	≤ 50	≤ 60
Cumulative Frequency	3	13	39	59	73	80

Draw a cumulative frequency curve.

Use your curve to estimate

- the median time to complete the crossword
 - the interquartile range.
- (c) The grouped frequency table below shows the minutes spent in a shopping complex by a number of people:

Minutes	5–15	15–25	25–35	35–65
Number of people	10	50	80	60

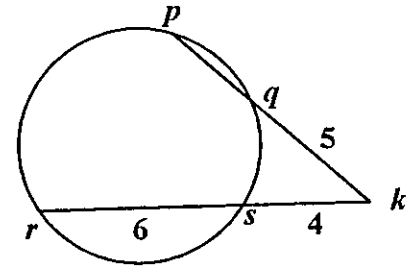
Note that 5–15 means that 5 is included but 15 is not, etc.

- Draw a histogram to illustrate the data.
- Calculate the mean number of minutes spent per person in the shopping complex, taking 10, 20 etc. as mid-interval values.

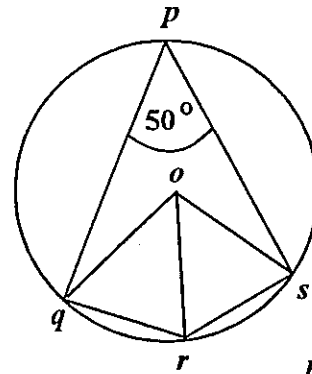
SECTION B

Attempt ONE question

8. (a) pq and rs intersect at k .
If $|rs| = 6$, $|sk| = 4$ and $|qk| = 5$, calculate $|pq|$.



- (b) Prove that the degree-measure of an angle subtended at a centre of the circle by a chord is equal to twice the degree-measure of any angle subtended by the chord at a point of the arc of the circle which is on the same side of the chordal line as is the centre.

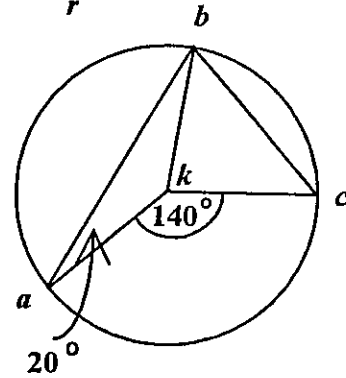


- (c) (i) The centre of the circle is o .
 $|\angle qps| = 50^\circ$ and $|\angle qor| = |\angle ros|$.
Find $|\angle qrs|$.

- (ii) The centre of the circle is k .
 $|\angle akc| = 140^\circ$ and $|\angle kab| = 20^\circ$.

Find $|\angle akb|$.

Find $|\angle kcb|$.

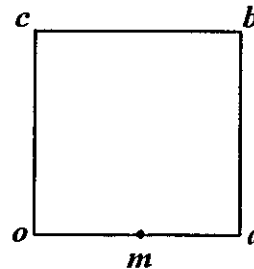


9. (a) $oabc$ is a square where o is the origin.
 m is the midpoint of $[oa]$.

Express in terms of \vec{a} and \vec{c}

(i) \vec{b}

(ii) \vec{cm} .



- (b) Let $\vec{p} = 3\vec{i} - 2\vec{j}$ and $\vec{q} = \vec{i} + 3\vec{j}$.

(i) Express \vec{pq} in terms of \vec{i} and \vec{j} .

(ii) Calculate $|\vec{pq}|$.

(iii) Find the value of the scalar k and the value of the scalar n for which

$$k(\vec{pq}) = (n + 3)\vec{i} + 2\vec{j}$$

- (c) Let $\vec{x} = 3\vec{i} + 5\vec{j}$ and $\vec{y} = 7\vec{i} + 4\vec{j}$.

Find $\vec{x}^\perp - \vec{y}^\perp$ in terms of \vec{i} and \vec{j} .

If $\vec{u} = \vec{x}^\perp$ and $\vec{v} = \vec{y}^\perp$, investigate if $vu \perp xy$.

10. (a) Expand $(1 + x)^4$ in ascending powers of x .

Show that $(1 + \sqrt{2})^4 = 17 + 12\sqrt{2}$.

(b) Find the sum to infinity of the geometric series

$$\frac{7}{10} + \frac{7}{100} + \frac{7}{1000} + \dots$$

Using this, show that

$$1.\dot{7} = \frac{16}{9}.$$

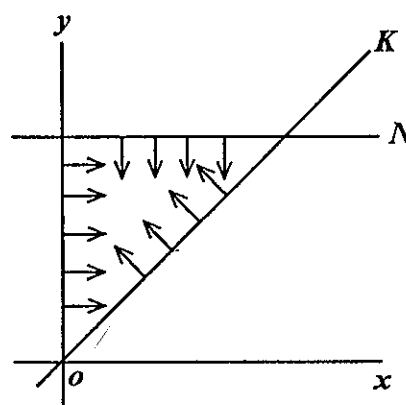
(c) A person invested IR£ x at the beginning of each year for 4 consecutive years at 10% per annum compound interest. The total value of the investments at the end of the fourth year was IR£51 051.

Find the value of x .

11. (a) The equation of the line K is $y - x = 0$ and the equation of the line N is $y - 4 = 0$.

(i) Write down the three inequalities which define the triangular region indicated in the diagram.

(ii) In a diagram, illustrate the set of points (x, y) that satisfy $y - 4 \geq 0$, $y - x \leq 0$ and $x - 6 \leq 0$.



(b) A property developer wishes to construct a business centre consisting of shops and offices. The floor space required for each shop is 60 m^2 and for each office is 20 m^2 . The total floor space for the business centre cannot exceed 960 m^2 .

The construction of each shop takes 5 working days to complete and each office 3 working days to complete. The developer has at most 120 working days to complete the construction.

(i) If the developer constructs x shops and y offices, write two inequalities in x and y and illustrate these on graph paper.

(ii) If the rental charge is IR£200 per m^2 for a shop and IR£140 per m^2 for an office, how many of each type should be built so as to maximize the developer's rental income? Find this maximum rental income.

(iii) If each shop provides 7 jobs and each office 3 jobs, write an expression in x and y for the total number of jobs to be provided. How many of each type should be built so as to maximize the number of jobs?