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LEAVING CERTIFICATE EXAMINATION, 1995

MATHEMATICS—ORDINARY LEVEL — PAPER 2 (300 marks)

59731

FRIDAY, 9 JUNE — MORNING, 9.30 to 12.00

Attempt 5 Questions from Section A and 1 Question from Section B. Each question carries 30 marks.
 Marks may be lost if necessary work is not shown or if you do not indicate where a calculator has been used.

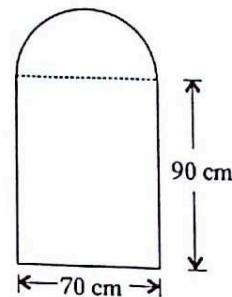


SECTION A

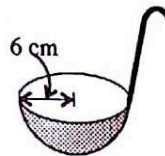
1. (a) A window is in the shape of a rectangle and a semicircle, as shown. The rectangular part of the window is 70 cm long and 90 cm high.

Find the area of the window in cm^2 .

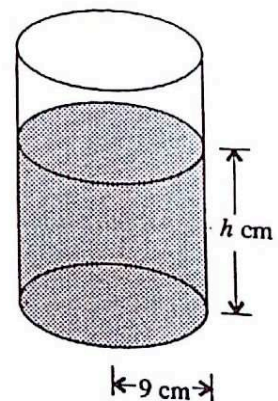
Take $\pi = \frac{22}{7}$.



- (b) A ladle in the shape of a hemisphere has a radius of length 6 cm. Find the capacity (internal volume) of the ladle in terms of π .



An empty cylinder has a radius of length 9 cm. When the ladle is filled with liquid and emptied into this cylinder 15 times, the liquid rises to a height of h cm in the cylinder, as shown.

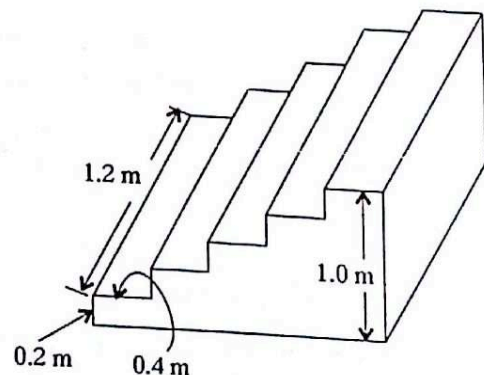


Calculate the value of h , correct to three significant figures.

- (c) Five rectangular shaped concrete steps are constructed as shown.

Each step measures 1.2 m by 0.4 m and the total height is 1.0 m with each step having the same height of 0.2 m.

Calculate the volume of this solid concrete construction.



OVER →

2. (a) The equation of the line L is $2x - y + 4 = 0$. L intersects the x axis at p and the y axis at q . Find the coordinates of p and the coordinates of q . Show L on a diagram.

The line K passes through the origin and is perpendicular to L .

Find the equation of K .

Find $K \cap L$.

Find the area of the triangle enclosed by L , K and the y axis.

- (b) $c(1,6)$ and $d(-3,-1)$ are two points. The point r has coordinates $(2, y)$ such that

$$|cd| = |cr|.$$

Find the two possible values of y .

3. (a) C is a circle centre $(0,0)$ and passing through the point $(3,1)$. Find

(i) the radius length

(ii) the equation of C .

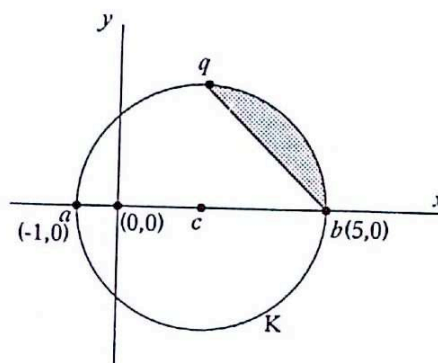
- (b) $a(-1,0)$ and $b(5,0)$ are the end points of a diameter of a circle K with centre c .

(i) Write down the coordinates of c and the radius length of K .

(ii) Find the equation of K .

(iii) T is a tangent to K and T is parallel to the x -axis. Find the two possible equations for T .

(iv) If $cq \perp ac$, where $q \in K$, find the area of the shaded region in terms of π .

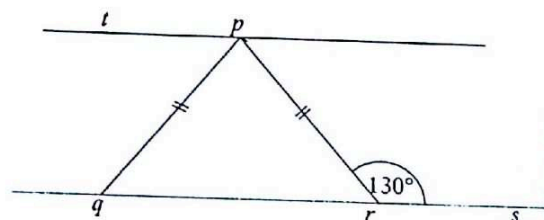


4. (a) $tp \parallel qr$, $|qp| = |pr|$ and $|\angle prs| = 130^\circ$.

Find

(i) $|\angle qpr|$

(ii) $|\angle qpt|$.



- (b) Prove that opposite sides of a parallelogram have equal lengths.

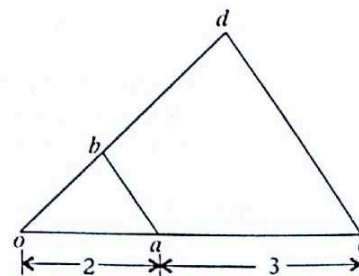
- (c) The triangle ocd is the image of the triangle oab under an enlargement, centre o , with $|oa| = 2$ and $|ac| = 3$.

(i) Find the scale factor of the enlargement.

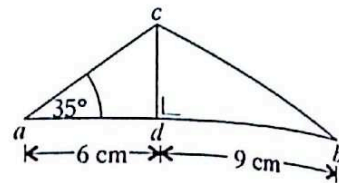
(ii) If $|ob| = 1.8$, find $|bd|$.

(iii) Calculate $|ab| : |cd|$.

(iv) If the area of the triangle ocd is 10.5 square units, find the area of the triangle oab .



5. (a) $|ad| = 6$ cm, $|db| = 9$ cm,
 $|\angle cad| = 35^\circ$ and $cd \perp ab$.
 Find, as accurately as the Tables allow, or
 by calculator,

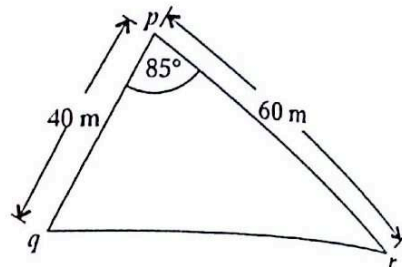


- (i) $|cd|$
 (ii) $|\angle cbd|$.

- (b) A plot of land has a triangular shape pqr ,
 as shown.

Find

- (i) the area of triangle pqr , correct to the
 nearest m^2
 (ii) $|qr|$, correct to the nearest metre.



- (c) $\cos A = -0.7914$. Find two values for A
 in $0 \leq A \leq 360^\circ$.

6. (a) One hundred students, girls and boys, took an examination and the grades achieved are recorded
 in the following table:

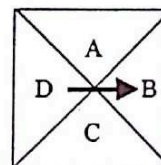
Grade	A	B	C	D	E
Number of Girls	8	20	18	10	4
Number of Boys	4	16	10	6	4

If a student is selected at random, what is the probability that the student is a

- (i) boy who got grade A
 (ii) boy or girl who got grade B
 (iii) girl who got grade A or grade B ?
- (b) How many different four-digit numbers can be formed from the digits 2, 4, 6 and 8, if no digit
 can be used more than once in a number?
 How many of these numbers
 (i) are greater than 6000?
 (ii) end with 8?

- (c) A game consists of spinning an unbiased arrow on a square board and throwing an unbiased die.

The board contains the letters A , B , C
 and D . The board is so designed that when
 the arrow stops spinning it can only point
 at one letter and it is equally likely to
 point at A or B or C or D .



List all possible outcomes of the game, that is, of spinning the arrow and throwing the die. Find
 the probability that in any one game the outcome will be

- (i) an A and a 6
 (ii) a B and an even number
 (iii) an A or a B and an odd number
 (iv) a C or a D and a number ≥ 4 .

7. (a) The table below shows the frequency of 0, 1, 2 or 3 goals scored in a number of football matches:

Number of goals scored	0	1	2	3
Number of matches	1	x	1	5

If the mean number of goals scored in a match is 2, find the value of x .

- (b) The intelligence quotients (I.Q.) of 50 children are recorded and grouped in the following table:

I.Q.	90—100	100—110	110—120	120—130
Number of children	11	22	12	5

Note that 90—100 means that 90 is included but 100 is not, etc.

Taking 95, 105 etc. as mid-interval values, calculate for this data

- the mean, correct to the nearest integer
 - the standard deviation, correct to the nearest integer.
- (c) The cumulative frequency table below shows the marks obtained by 100 students in a school test.

Marks	≤ 20	≤ 40	≤ 60	≤ 80	≤ 100
Cumulative Frequency	4	16	70	91	100

Draw a cumulative frequency curve.

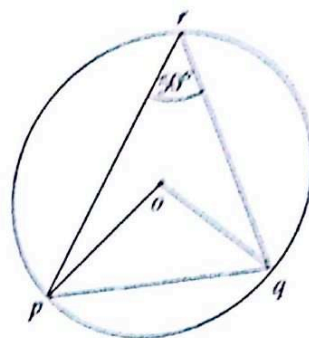
- Use the curve to estimate the number of students who got less than 50 marks.
- The school decides that the 15 highest marked students will each receive a prize.
Use the curve to estimate the least mark a student must obtain in order to qualify for a prize.

SECTION B

Attempt ONE question

8. (a) The centre of the circle is o and $|\angle prq| = 50^\circ$.
Find

- (i) $|\angle poq|$
(ii) $|\angle opq|$.



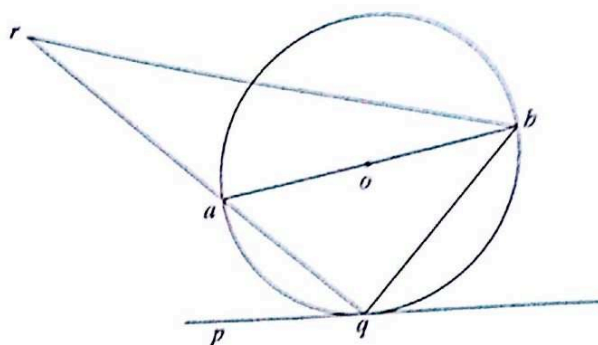
- (b) Prove that an angle between a tangent ck and a chord $[cd]$ of a circle has degree-measure equal to that of any angle in the alternate segment.

- (c) $[ab]$ is a diameter of the circle with centre o . The line pq is a tangent to the circle at q .

The point r on qa is joined to b .

If $|\angle aqp| = 40^\circ$ and $|\angle arb| = 22^\circ$, find

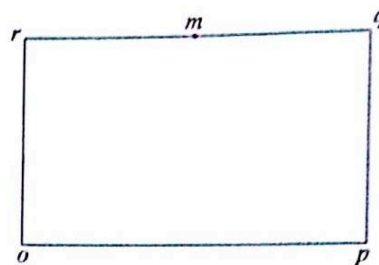
- (i) $|\angle abq|$
(ii) $|\angle qab|$
(iii) $|\angle abr|$.



9. (a) $pqro$ is a rectangle where o is the origin. The midpoint of $[rq]$ is m .

Express in terms of \vec{p} and \vec{r}

- (i) \vec{q}
(ii) \vec{m}
(iii) \vec{mp} .



- (b) Let $\vec{x} = 3\vec{i} - 2\vec{j}$ and $\vec{y} = 5\vec{i} + 8\vec{j}$.

(i) Express $\vec{x} + \vec{y}$ in terms of \vec{i} and \vec{j} .

(ii) Calculate $|\vec{x} + \vec{y}|$.

(iii) Find the value of the scalar k and the value of the scalar n for which

$$k(\vec{x} + \vec{y}) = 16\vec{i} + (n - 2)\vec{j}.$$

(iv) Investigate whether or not $\vec{x} \perp \vec{y}$.

(v) Determine if

$$(\vec{x} + \vec{y})^\perp = \vec{x}^\perp + \vec{y}^\perp.$$

10. (a) A pupil saves money each day in the month of November. The pupil saves 10p on the 1st of November, 15p on the 2nd., 20p on the 3rd., continuing this pattern until the last day of November.
How much will the pupil have saved at the end of the 30 days?

- (b) (i) Find the sum to infinity of the geometric series

$$\frac{3}{100} + \frac{3}{1000} + \frac{3}{10\,000} + \dots$$

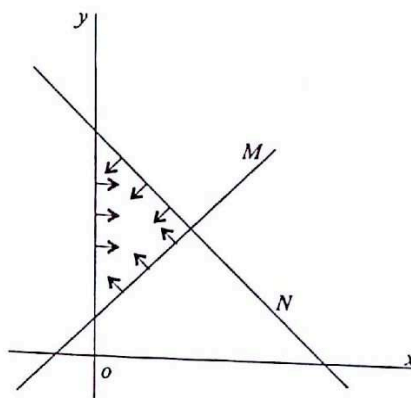
- (ii) Using an infinite series, show that

$$0.1\dot{3} = \frac{2}{15}.$$

- (c) Expand $(1 + x)^6$ in ascending powers of x . Find, using the expansion, the value of $(1 + \sqrt{5})^6$ in the form $a + b\sqrt{5}$, where $a, b \in \mathbf{Z}$.
Show that

$$(1 + x)^6 - (1 - x)^6 = 4x(3 + 10x^2 + 3x^4).$$

11. (a) The equation of the line M is $x - y + 1 = 0$ and the equation of the line N is $x + y - 6 = 0$.
Write down the three inequalities which define the triangular region indicated in the diagram.



- (b) A farmer has not more than 2000 m^2 of ground for planting apple trees and blackcurrant bushes. The ground space required for an apple tree is 50 m^2 and for a blackcurrant bush is 5 m^2 . The planting of an apple tree costs IR£20 and the planting of a blackcurrant bush costs IR£4. The farmer has at most IR£1000 to spend on planting.
If the farmer plants x apple trees and y blackcurrant bushes, write two inequalities in x and y and illustrate these on graph paper.
When fully grown, each apple tree will produce a crop worth IR£90 and each blackcurrant bush a crop worth IR£15.
How many of each should be planted so that the farmer's gross income is a maximum?
Calculate the farmer's maximum profit.