1. (a) A prize fund of IR£6000 is divided as follows:

   > the first prize is half the fund,
   > the second prize is two-thirds the first prize,
   > the third prize is what remains.

   How much is the third prize worth?

   (b) IR£5000 is invested for two years at compound interest.

   (i) The interest at the end of the first year was IR£275.
        Calculate the rate of interest for the year.

   (ii) At the end of the second year the investment was worth IR£5644.25.
        Calculate the rate of interest for the second year.

   (c) Members of a club had two weeks to raise money for their club.
       A business person agreed to give 50p to the club for every IR£5 collected by the members.

   (i) In the first week the members collected IR2640. How much did the
        business person give the club in the first week?

   (ii) In the second week the money raised from the members and the business
        person amounted to IR£6930.
        How much did the club members collect in the second week?

2. (a) Show that $x = 5$ is a root of the equation

   $$x^3 - x^2 - 17x - 15 = 0.$$

   (b) Find the solution set E of $2x + 7 \le 19$, $x \in \mathbf{R}$.
   Find the solution set H of $3 - 2x \le 11$, $x \in \mathbf{R}$.
   Find $E \cap H$.

   (c) If $x + \dfrac{1}{x} = a$, show that $x^2 + \dfrac{1}{x^2} = a^2 - 2$.

   Express $x^3 + \dfrac{1}{x^3}$ in terms of a.

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3. (a) Solve for x the equation

$$3^{2x} = 9 .$$

(b) Solve for x and y

$$x + 2y = 3$$
$$x^2 + y^2 = 26.$$

(c) Let $f(x) = x^2 + bx + c, \ x \in R.$

The solutions of $f(x) = 0$ are -3 and $1.$
Find the value of b and the value of $c.$

If $f(-1) = k$, find the value of $k.$

Solve the equation

$$f(x) - k = 0.$$

4. (a) Let $z_1 = 5 + 4i$ and $z_2 = -3 - 5i$, where $i^2 = -1.$

Plot

(i) z_1
(ii) z_2
(iii) $z_1 + z_2$

on an Argand diagram.

(b) Let $w = \dfrac{1 + i}{2 - 2i} .$

Express w in the form $p + qi, \ p, q \in \mathbf{R} .$
Calculate $|w|.$

Verify that

$$|w|^2 = w\overline{w} ,$$

where $\overline{w}$ is the complex conjugate of $w.$

(c) (i)   Let $u = 6 - 5i.$
Solve for real a and real b

$$u + ai = 2b.$$

(ii)   Solve for real s and real t

$$s(2 - i) + ti(4 + 2i) = 1 + s + ti.$$

If $z = x + iy$, where $x, y \in \mathbf{R}$, what type of curve is represented by

$$|z|^2 = |s + it|^2 \ ?$$

5. (a) The first two terms of an arithmetic sequence are 5, 0, ...

   Find

   (i) d, the common difference
   (ii) T_{11}, the eleventh term.

   (b) The numbers

$$\frac{1}{2}, \ 1, \ 2, \ .....$$
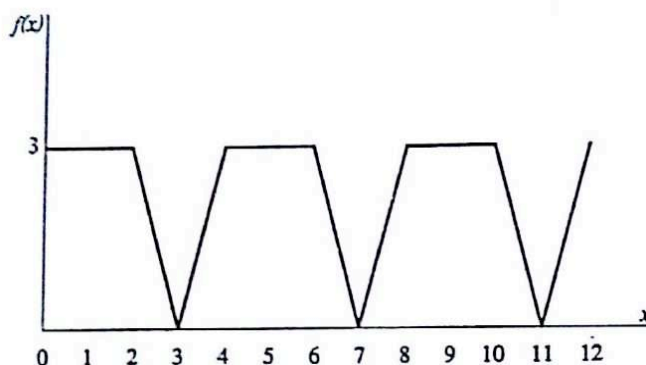
   are in geomteric sequence.

   Find

   (i) r, the common ratio
   (ii) T_n, the nth term
   (iii) S_6, the sum to six terms.

   (c) In an arithmetic series, the tenth term, T_{10}, is 19 and the sum to ten terms, S_{10}, is 55.
   Find the first term and the common difference.
   Show that

$$2S_n = 3n^2 - 19n.$$

6. (a) The graph shows portion of
   a periodic function $f : x \rightarrow f(x)$.
   Write down the period and
   range of the function.

   (b) Differentiate from first principles

$$x^2 - 6x + 4$$

   with respect to x.

   (c) Let $f(x) = 2x^3 - 7x^2 + 7x - 2$, for $x \in \mathbf{R}$.

   Find the derivative of $f(x)$.

   Find the equation of one of the tangents to the curve of $f(x)$ which is parallel to the line
   $y = 3x - 8$.

7. **(a)** Differentiate

$$6 - 5x^3$$

with respect to x.

(b) **(i)** Differentiate

$$(2x - 3)^7$$

with respect to x.

(ii) Let $y = \dfrac{5 + x^2}{2 - x}$, $x \neq 2$.

Find the values of x for which $\dfrac{dy}{dx} = 0$.

(c) The air resistance R to a body moving with speed v metres per second is given by

$$R = \dfrac{v^2}{100}.$$

Find the rate of change of the air resistance with respect to the speed. Calculate this rate of change when $v = 16$ m/s.

8. **(a)** If $s = t^3 - 4t^2$, find $\dfrac{ds}{dt}$ when $t = 3$.

(b) Let $f(x) = \dfrac{1}{x - 3}$, for $x \in \mathbf{R}$ and $x \neq 3$.

(i) Find $f(0)$, $f(\frac{8}{3})$, $f(\frac{10}{3})$ and $f(6)$.

(ii) Find $f'(x)$, the derivative of $f(x)$.

(iii) Draw the graph of

$$f(x) = \dfrac{1}{x - 3}$$

for $0 \leq x \leq 6$.

(iv) Solve the equation

$$f(x) + f'(x) = 0.$$