AN ROINN OIDEACHAIS

M. 27

LEAVING CERTIFICATE EXAMINATION, 1994

MATHEMATICS — ORDINARY LEVEL — PAPER I (300 marks)

THURSDAY, 9 JUNE — MORNING, 9·30 to 12·00

Attempt **SIX QUESTIONS** (50 marks each)

Marks may be lost if all your work is not clearly shown or if you do not indicate where a calculator has been used.

1. **(a)** A person buys 490 German Marks when the exchange rate is IR£1 = 2.45 Marks.

A charge is made for this service.

How much, in IR£, is this charge if the person pays IR£205.50?

(b) Write

$$2.3 \times 10^{-2} + 3.5 \times 10^{-3}$$

as a decimal number.

Say if this number is greater than or less than 0.02.

(c) A person earns IR£19 400 per annum and has tax-free allowances of IR£8600.

Tax is paid at the rate of 27% on the first IR£7700 of taxable income and at the rate of 48% on the remainder of taxable income.

Calculate the yearly take home pay.

By how much is the yearly take home pay increased if the 27% rate of tax is reduced to 25%?

2. **(a)** Solve the inequality

$$3x - 7 < 2, \quad x \in \mathbf{R}$$

and indicate the solution set on the number line.

(b) Solve for x and y

$$y = x + 2$$
$$x^2 + y^2 = 10$$

(c) Write $\frac{81}{\sqrt{3}}$ as a power of 3, and solve for x the equation

$$3^{x-2} = \left(\frac{81}{\sqrt{3}}\right)^2.$$

OVER →

3. **(a)** Show that $x = -3$ is a root of the equation

$$x^3 - x^2 - 8x + 12 = 0.$$

(b) Express t in terms of p and q when

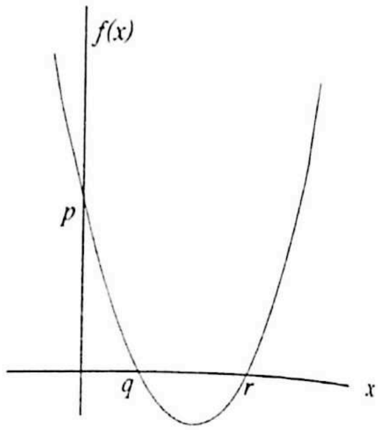
$$p = \frac{q - t}{3t}, \quad t \neq 0.$$

Calculate the value of t when $p = 0.5$ and $q = 25$.

(c) The graph of the quadratic function

$$f : x \rightarrow x^2 - 4x + 3,\ x \in \mathbf{R}$$

cuts the axes at p, q and r as shown.



Find the coordinates for each of the points p, q and r.

Write down the expression for $g(x)$ where

$$g(x) = f(-x).$$

Show that there are no real solutions to the equation

$$f(x) + g(x) = 0.$$

4. **(a)** Simplify

$$2 + 3i(4 + 5i) - 6i$$

and express your answer in the form $p + qi$, where $p, q \in \mathbf{R}$ and $i^2 = -1$.

(b) Let $z = (1 - 2i)(3 - i)$.

Plot z, $z + 3$ and $\bar{z}$ on an Argand diagram, where $\bar{z}$ denotes the complex conjugate of z.

Calculate $|z\bar{z}|$.

(c) Let $w = 3 - 4i$.

(i) Solve for real x and real y

$$x + w = 3yi.$$

(ii) Solve for real s and real t

$$|w|(s + it) = \frac{5}{\bar{w}},$$

where $\bar{w}$ denotes the complex conjugate of w.

5. (*a*) The n th term, T_n, of an arithmetic sequence is

$$T_n = 52 - 4n.$$

Find

(i) T_1, the first term
(ii) d, the common difference
(iii) the term which is zero
(iv) the sum of the terms which are positive.

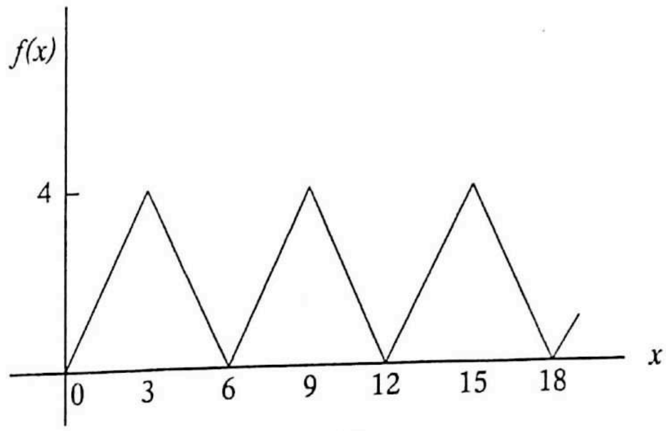
(*b*) The first two terms of a geometric series are

$$32 + 16 + \ldots\ldots$$

Find

(i) r, the common ratio
(ii) T_n, the n th term
(iii) S_n, the sum to n terms
(iv) the value of $S_n + T_n$ when $n = 4$.

6. (a) The graph shows portion of a periodic function $f : x \rightarrow f(x)$.
Write down the period and range of the function.



(b) Find, using calculus, the coordinates of the local maximum of the curve

$$y = 6x^2 - x^3,$$

given that the curve has a local minimum at (0,0).

(c) Draw the graph of the function

$$f : x \rightarrow 6x^2 - x^3$$

in the domain $-2 \leq x \leq 6,\ x \in \mathbf{R}$,

given that $f(4) = 32$, $f(5) = 25$ and $f(6) = 0$.

Find the solution set for which $f(x)$ is decreasing.

OVER →

7. **(a)** Differentiate

$$x(5 - 3x^2)$$

with respect to x.

(b) **(i)** Find the value of $\frac{dy}{dx}$ at $x = 2$ when $y = (1 - x^2)^3$

(ii) Find $\frac{dy}{dx}$ when $y = \frac{1 - x^2}{x}$.

Show that $\frac{dy}{dx} < 0$ for all $x \neq 0,\ x \in \mathbf{R}$.

(c) The height h metres of a balloon is related to the time t seconds by

$$h = 120t - 15t^2.$$

Find

(i) its height after 2 seconds
(ii) the maximum height reached by the balloon.

8. **(a)** The funtion f is defined by

$$f : \mathbf{R} \rightarrow \mathbf{R} : x \rightarrow 4x - 5.$$

Find $f(3)$.

Hence find the value of k for which

$$k\,f(3) = f(10).$$

(b) Let $g(x) = \frac{1}{x}$, for $x \in \mathbf{R}$ and $x \neq 0$.

Find $g(¼)$, $g(½)$, $g(1)$, $g(2)$, $g(4)$.

Under the central symmetry in the origin, find the image of each of the points (1, 1) and (4, ¼).

(c) Using the information obtained in **(b)**, draw the graph of

$$g(x) = \frac{1}{x}$$

for $-4 \leq x \leq 4$.

Find the derivative of $g(x)$.

Prove that the tangents to $g(x)$ at (1, 1) and (–1, –1) are parallel to each other.