AN ROINN OIDEACHAIS

M. 28

47161

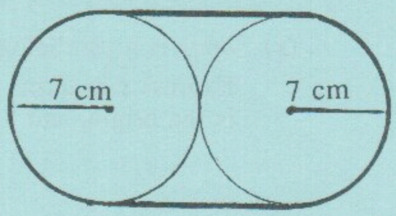
LEAVING CERTIFICATE EXAMINATION, 1993

MATHEMATICS — ORDINARY LEVEL — PAPER II

FRIDAY, 11 JUNE – MORNING, 9·30 to 12·00

Attempt **QUESTION 1** (100 marks) and **FOUR** other questions (50 marks each)
Marks may be lost if necessary work is not clearly shown or if you do not indicate where a calculator has been used.

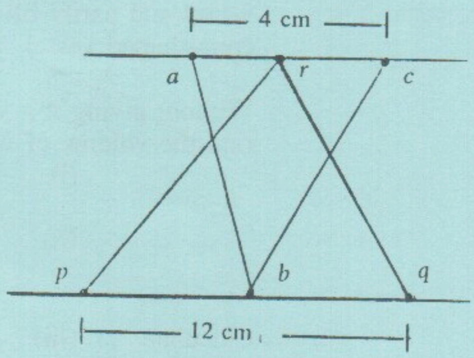
1. *(i)* A belt passes round two touching circular wheels, each of radius length 7 cm.
Calculate the length of the belt.
Take $\pi = \frac{22}{7}$.



(ii) Express q in terms of p and r, if $p = \sqrt{2q - r}$.

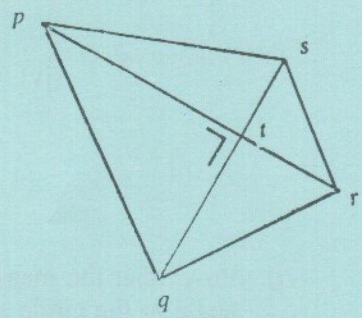
(iii) The two triangles pqr, abc are between parallel lines.
Find the value of

$$\frac{\text{area of the triangle } abc}{\text{area of the triangle } pqr}.$$

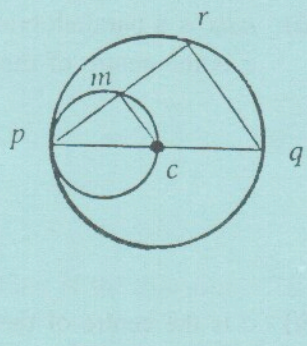


(iv) $pqrs$ is a quadrilateral in which $pr \perp qs$. Prove

$$|pq|^2 + |rs|^2 = |qr|^2 + |ps|^2.$$



(v) $[pq]$ is a diameter of the circle.
$[pc]$ is a diameter of the smaller circle.
Prove $mc \parallel rq$.



(vi) $pqrs$, in that order, is a parallelogram in which the coordinates $p(-3, -2)$, $q(1, -1)$ and $r(3, 5)$ are known. Find the coordinates of s.

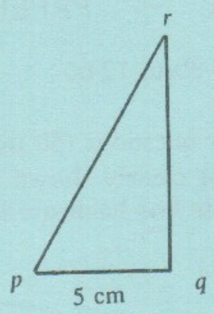
OVER →

1. contd.

(vii) The point q (3, k) is on the line $2x - 3y = 12$. p is the point (–3, –6). Calculate the mid-point of [pq].

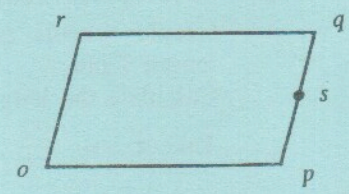
(viii) Find the points of intersection of the circle $x^2 + y^2 + 9x - 16 = 0$ and the y-axis.

(ix)



In the triangle pqr, $pq \perp qr$ and $|pq| = 5$ cm. If $\cos |\angle rpq| = 0{\cdot}4$, calculate $|pr|$.

(x) $opqr$ is a parallelogram. Express $\vec{s}$ in terms of $\vec{p}$ and $\vec{r}$, if $\vec{o}$ is the origin, and $|ps| = |sq|$.



2. Water which just filled a cylinder was used to just fill an upright cone-shaped vessel and partly fill another identical cone-shaped vessel (see diagram).

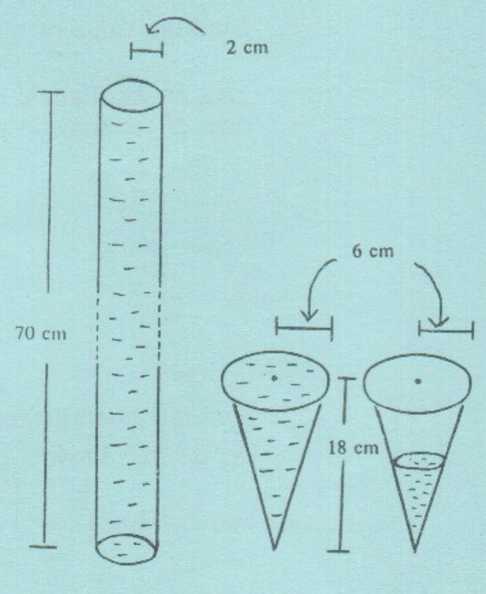
Without giving π a value, find the volume of water which just filled

(i) the cylinder

(ii) the first upright, cone-shaped vessel.

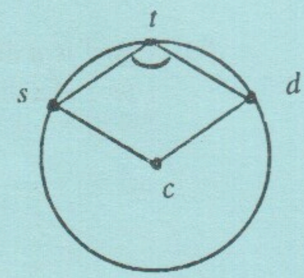
Calculate (iii) the height of water left in the cylinder after the first cone-shaped vessel was filled.

(iv) the height of water in the second cone-shaped vessel.

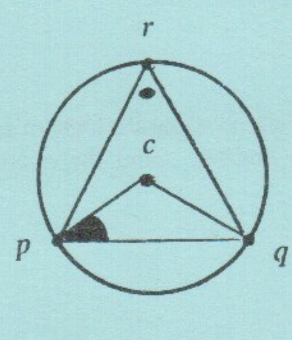


3. *(i)* Prove that the measure of the angle at the centre of a circle is twice the measure of an angle at the circle standing on the same arc.

(ii) $cdts$ is a parallelogram and c is the centre of the circle. Prove $|\angle dts| = 120°$.



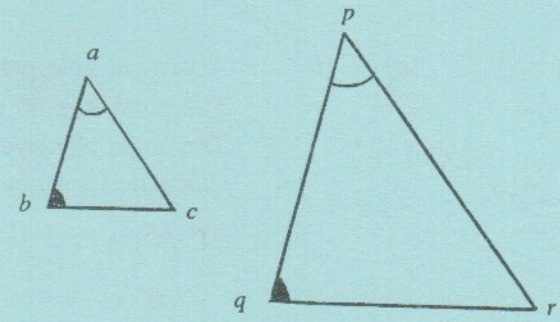
(iii) c is the centre of the circle. Prove, $|\angle cpq| + |\angle qrp| = 90°$



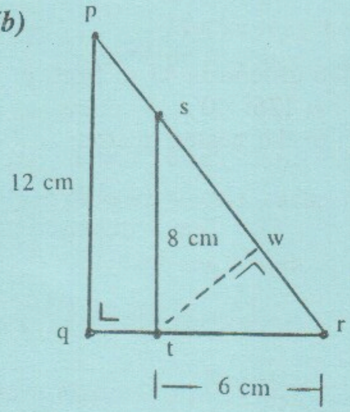
4.

(a) The corresponding angles of two triangles are equal in measure, as shown.

Prove that the corresponding sides of the two triangles are in proportion.



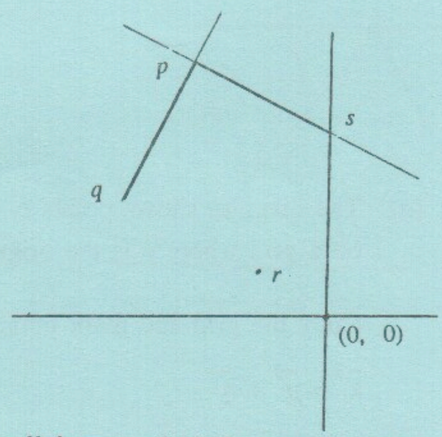
(b)



(i) In the triangle pqr , $pq \perp qr$ and $st \parallel pq$.
Calculate $|qt|$, if $|pq| = 12$ cm, $|st| = 8$ cm and $|rt| = 6$ cm.

(ii) If $tw \perp pr$, calculate $|wr|$.

5.



ps is the line $x + 3y - 12 = 0$
pq is the line $3x - y + 14 = 0$.

Calculate the coordinates of

(i) p.

(ii) s, the point of intersection of ps and the y-axis.

Given that r is the point $(-1, 1)$ and that $pqrs$ is a parallelogram, find the equation of

(iii) rp

(iv) sq.

(v) Verify that $pqrs$ is a square.

6.

$S_1 : x^2 + y^2 = 25$ is a circle.
Write down the length of its radius.

S_2 is the image of S_1 under a translation $(0, 0) \to (-3, 4)$. What is the equation of S_2?

T is the tangent at $(4, 3)$ to S_1. What is the equation of T?

Verify that the line $4x + 3y = 25$ is a tangent to S_2 and find the coordinates of the point of tangency.

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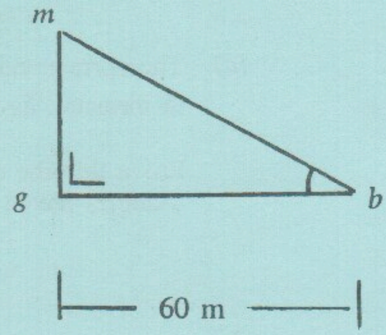
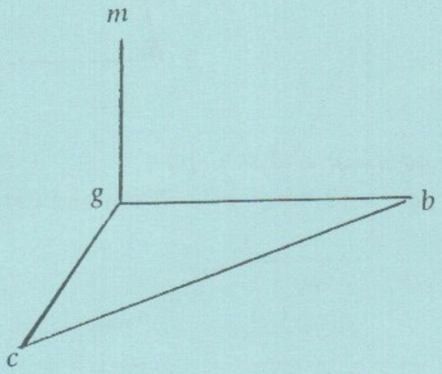
7. (a) (i) A vertical pole gm stands on level ground.
The point b is 60 m from the base of the pole.
The angle of elevation $|\angle gbm| = 34° \ 15'$ (or $34{\cdot}25°$).
Calculate $|gm|$ correct to two places of decimals.



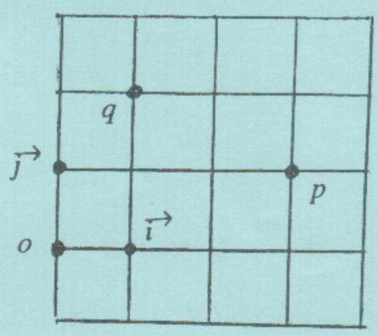


(ii) A point c on the ground is 45 m from g and $|\angle bgc| = 126° \ 40'$.
Calculate $|bc|$ to the nearest metre.

(b) Sketch the graph of $2\cos x$ in the domain $-\pi \leq x \leq \pi$.

Use the graph to estimate the range of values of x for which $\cos x < \frac{3}{4}$.

8. (a) The diagram shows points o, p, q on a grid where o is the origin.



Copy the grid and mark points k_1 and k_2 such that

$\vec{k_1} = \vec{p} + \vec{q}$

$\vec{k_2} = \vec{p} - \vec{q}$.

If $\vec{q} = \vec{i} + 2\vec{j}$, express $\vec{p}$, $\vec{k_1}$, $\vec{k_2}$, in terms of $\vec{i}$, $\vec{j}$.

Verify $\vec{p} = \frac{1}{2}(\vec{k_1} + \vec{k_2})$.

(b) The diagram shows a triangle opq with r the mid-point of $[pq]$.
$|ot| = \frac{3}{4}|or|$.

If o is the origin, express

$\vec{to} + \vec{tp} + \vec{tq}$

in terms of $\vec{p}$ and $\vec{q}$.

If $\vec{r} = 4\vec{i} + 4\vec{j}$, show that

$\vec{to} + \vec{tp} + \vec{tq} = -(\vec{i} + \vec{j})$.