AN ROINN OIDEACHAIS **M. 27**

LEAVING CERTIFICATE EXAMINATION, 1993

MATHEMATICS — ORDINARY LEVEL — PAPER I (300 marks)

THURSDAY, 10 JUNE — MORNING, 9.30 to 12.00

Attempt **QUESTION 1** (100 marks) and **FOUR** other questions (50 marks each)
Marks may be lost if all your work is not clearly shown or if you do not indicate where a calculator has been used.

1. (i) A shopkeeper bought a consignment of 1000 pears for IR£120.
The shopkeeper sold 90% of them in bags of 6 for IR£1.10 per bag.
The remaining 10% were discarded.
Calculate the shopkeeper's profit.

(ii) A rectangular hall measures 52 m in length and 40 m in width. During a sale of work, $\frac{9}{13}$ of the area of the hall is taken up by stalls. The remainder of the area is used up by passages.
Calculate the area of the passages.

(iii) Solve

$$(x + 1)^2 = 4.$$

(iv) Show that $x = -3$ is a root of

$$2x^3 + x^2 - 13x + 6 = 0.$$

(v) Solve the simultaneous equations

$$x = 2y$$

$$2(x + 3) = 5 + 3y.$$

(vi) The numbers

$$x, \; 48, \; -24$$

are in geometric sequence. Verify that the common ratio of this sequence is $-\frac{1}{2}$.
Hence find x.

(vii) A combination lock consists of three rings each marked with the digits 0, 1, 2, ..., 9.
There is only one correct combination to open the lock.
What is the maximum number of different unsuccessful attempts that could be made to open the lock ?

(viii) Shade, in a diagram, the triangular region enclosed by the three lines
$y = x$, $y = -x$ and $y = 2$ for $x, y \in \mathbf{R}$.

(ix) The function f is defined as

$$f : \mathbf{R} \to \mathbf{R} : x \to 5 - 3x.$$

Find $f(2)$.

Find the two values of x for which

$$x f(x) = 0.$$

(x) Differentiate $\dfrac{2x}{3 - x^2}$ with respect to x.

2. Let $z = 1 + i$ and $z_1 = 1 - i$, where $i = \sqrt{-1}$.
Show that z is a solution of the equation

$$z^2 - 2z + 2 = 0$$

Plot z and z_1 on an Argand diagram.
Calculate u if

$$u = \frac{z}{z_1}.$$

Plot u on an Argand diagram.
Write down w, the image of u under the central symmetry in z_1. Calculate $| u - w |$.

3. Eighty employees in a company were surveyed to find how long each spent on a morning tea-break. The result of the survey showed:

Time in minutes	0 – 4	4 – 8	8 – 12	12 – 16	16 – 20
Number of employees	10	30	15	20	5

(Note: 0 - 4 means 0 is included but 4 is not, etc.)

What is the largest possible number of employees who took no tea-break?.
Calculate the mean time of a tea-break, taking the times at the mid-interval values.

Complete the corresponding cumulative frequency table:

Time in minutes	<4	<8	<12	<16	<20
Number of employees					

Draw the cumulative frequency curve.

Use the curve to estimate

(i) the number of employees who spent less than the mean time on their tea-break.

(ii) the percentage of employees who were within 1.5 standard deviations of the mean, given that the standard deviation from the mean is 5 minutes.

4. (a) If $f(x) = x^3 + 3x^2 - 2$, complete the following table:

x	-3	-2	-1	0	1	$1{\cdot}5$
$f(x)$						$7{\cdot}825$

Draw the graph of the function:

$$f : x \rightarrow x^3 + 3x^2 - 2$$

in the domain $-3 \leq x \leq 1{\cdot}5$, for $x \in \mathbf{R}$.

Use your graph to estimate the values of x for which $f(x) = -1$.

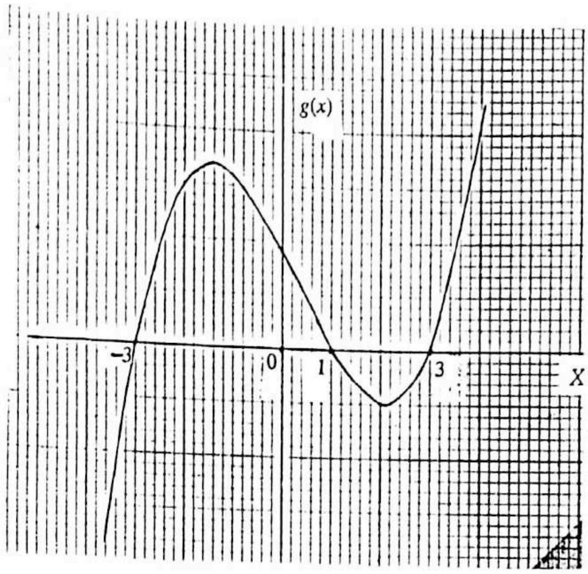
Use your graph, or otherwise, to estimate the values of x for which

$$x^3 + 3x^2 - 3 = 0.$$

(b) The diagram shows a graph (sketch) of the function $x \rightarrow g(x)$ where $g(x)$ is a cubic expression and the coefficient of x^3 is 1.
This graph cuts the X axis at $x = -3$, $x = 1$ and $x = 3$.

(i) Find $g(x)$

(ii) Find $g(0)$



5. (a) Show that

$$\frac{x}{x-1} - 1$$

can be expressed as the single fraction $\frac{1}{x-1}$.
Hence, or otherwise, solve

$$\frac{x}{x-1} - 1 = \frac{x+1}{2}$$

(b) Find the value of x if

$$7^{x-1} = \sqrt{7}$$

(c) Write out and simplify the terms of the expansion of

$$(1 - 2x)^3$$

in ascending powers of x.

Find a value of x for which

$$(1 - 2x)^3 - 2x^2(6 - 4x) = 0$$

6. **(a)** The general term, T_n, in an arithmetic sequence is

$$T_n = 3 - 4n.$$

Calculate

(i) T_1, the first term

(ii) d, the common difference

(iii) S_{10}, the sum of the first 10 terms.

Verify that

$$S_{10} - S_9 = T_{10}.$$

(b) A person borrows IR£5000 at 12% per annum compound interest.
How much will the person repay if the loan is to be repaid in three instalments as follows:
the first instalment of IR£1000 at the end of the first year,
the second instalment of IR£2000 at the end of the second year and the remainder at the end of the third year.

7. A train consists of two different types of carriages - first class carriages and standard class carriages. The total number of carriages cannot exceed 10. There must be at least one first class carriage.

Each first class carriage can seat 30 people. Each standard class carriage can seat 75 people. The train must have a minimum of 570 seats.

Graph the set showing the possible number of each type of carriage.

What is the maximum number of people who could be seated on the train ?

The profit from each first class carriage when full is IR£300 and from each standard class carriage when full is IR£200. Calculate the maximum profit possible.

8. **(a)** Differentiate from first principles $x^2 - 2x$ with respect to x .

(b) Find the value of $\frac{dy}{dx}$ at $x = 0$ and at $x = 1$ when

$$y = (2x^2 + 3)(x^2 - x + 1).$$

Hence, or otherwise, verify that the graph of this function has a local maximum or a local minimum between $x = 0$ and $x = 1$.

(c) The volume V of water flowing over a weir per minute is given by

$$V = 300h^{3/2}$$

where h is the height of the water above the weir.
Find the rate of change of V with respect to h.
Calculate this rate of change when $h = 4$.