AN ROINN OIDEACHAIS **M. 27**

LEAVING CERTIFICATE EXAMINATION, 1992

# MATHEMATICS — ORDINARY LEVEL — PAPER I ( 300 marks )

55835

THURSDAY, 11 JUNE — MORNING, 9·30 to 12·00

Attempt **QUESTION 1** (100 marks) and **FOUR** other questions (50 marks each)

**Marks may be lost if all your work is not clearly shown**
**or if you do not indicate where a calculator has been used.**



1. **(i)** A rock concert was attended by 15 000 persons.
Each person paid IR£12 to attend the concert.
The total costs for organising the concert were IR£125 000.
Calculate the profit the organisers made as a percentage of the total costs.

**(ii)** A person on holiday in America buys \$320 when the exchange rate is **IR£1 = \$1.60**.
A charge of 2·5% is made for this service.
How much does the person pay in IR£ ?

**(iii)** Solve

$$2x^2 + 3(4 - x) = 14x - 9.$$

**(iv)** If $x = -1$ is a solution of the equation

$$2x^3 - x^2 + tx + 1 = 0,$$

find the value of $t$.

**(v)** Find $a$, the first term, and $r$, the common ratio, of the geometric sequence where

$$T_2 = 21 \text{ and } T_3 = -63$$

are the second and third terms, respectively.

**(vi)** Solve the simultaneous equations

$$3x + 4y = 12$$
$$2x + 3y = -7$$

**(vii)** If $3^{x-2} = \sqrt{27}$, find the value of $x$.

**(viii)** Shade in a diagram the region enclosed by the X axis, the Y axis
and the line $x + 2y = 4$,
for $x, y \in \mathbf{R}$.

**(ix)** The function $f$ is defined as

$$f : \mathbf{R} \to \mathbf{R} : x \to 4x - 8.$$

Find $f(3)$.

Hence find the value of $k$ for which

$$k f(3) = f(6).$$

**(x)** Differentiate $\dfrac{x^3}{4x - 7}$ with respect to $x$.

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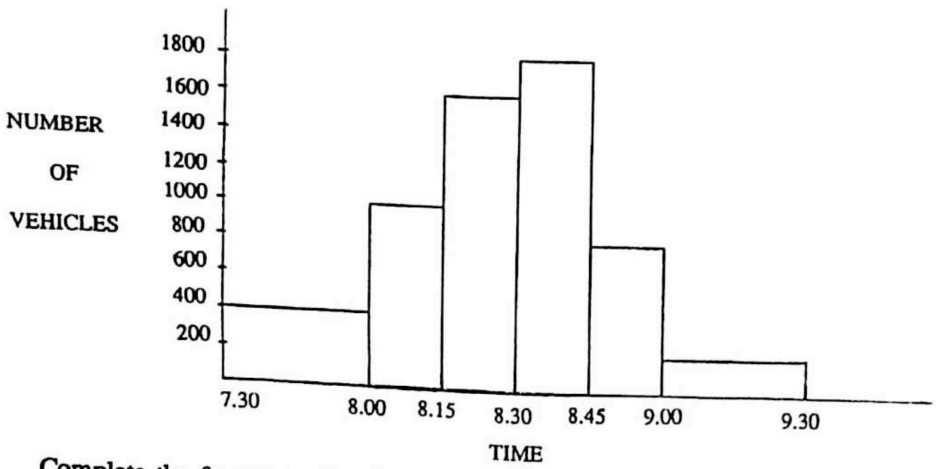
2. Let $z_1 = 4 - 2i$ and $z_2 = 4 + 2i$, where $i = \sqrt{-1}$.
Plot $z_1$ and $z_2$ on an Argand diagram.
Investigate if $z_1 . z_2 = |z_1|^2$.

If $w$ is the image of $z_1$ under the central symmetry in $z_2$, express $w$ in the form $a + bi$.
Calculate $|w - z_2|$.

Draw the set K of all complex numbers such that each is a distance of 4 from $z_2$.

Find $u$ and $v$, complex numbers of K, such that the four points $w$, $u$, $z_1$ and $v$ form the vertices of a square.
Write your answers in the form $a + ib$.

3. (a) A survey of the traffic flow through a busy junction began at 7.30 in the morning and concluded two hours later. The histogram illustrates the results for the survey.




Complete the frequency distribution table below for this histogram:

| Time interval | 7.30 - 8.00 | 8.00 - 8.15 | 8.15 - 8.30 | 8.30 - 8.45 | 8.45 - 9:00 | 9.00 - 9.30 |
|---|---|---|---|---|---|---|
| Number of vehicles | | | 1600 | | | |

In which time interval do exactly 1000 vehicles pass through the junction ?

(b) At a sports meeting, the distances, measured in metres, for 50 competitors each throwing the javelin once are given as follows:

| Distance (m) | 30 - 40 | 40 - 50 | 50 - 60 | 60 - 70 | 70 - 90 |
|---|---|---|---|---|---|
| No. of competitors | 6 | 12 | 16 | 12 | 4 |

(Note: 30 — 40 means 30 is included but 40 is not, etc.)

Complete the cumulative frequency table below:

| Distance (m) | < 40 | < 50 | < 60 | < 70 | < 90 |
|---|---|---|---|---|---|
| No. of competitors | | | | | |

Draw the cumulative frequency curve.

Use this curve to estimate

(i) the number of competitors who threw the javelin between 45 m and 65 m.
(ii) the range of distances thrown by the worst 16 competitors.

4. If $f(x) = x^3 - 3x + 1$, complete the following table:

| $x$ | $-3$ | $-2$ | $-1$ | $0$ | $1$ | $2$ |
|---|---|---|---|---|---|---|
| $f(x)$ | $-17$ | | | | | |

Draw the graph of the function:

$$f : x \rightarrow x^3 - 3x + 1$$

in the domain $-3 \leq x \leq 2$, for $x \in \mathbf{R}$.

Use your graph to estimate the range of values of $x$ for which the tangents to the graph have negative slope.

Using the same axes and the same scales draw the graph of the function

$$g : x \rightarrow 1 - x, \text{ for } x \in \mathbf{R}.$$

Write down the values of $x$ at which the two graphs intersect.

Solve algebraically

$$x^3 - 3x + 1 = 1 - x.$$

5. (a) Express $$\frac{x - 1}{x + 1} - 1$$

as a single fraction.

Hence, or otherwise, solve

$$\frac{x - 1}{x + 1} - 1 = \frac{1}{x - 1}.$$

(b) If

$$5! + 6! = k\,5!,$$

find $k$.

(c) Write out and simplify the first four terms of the expansion of

$$(1 - 2x)^5$$

in ascending powers of $x$.

When $x = \frac{1}{2}$, find the value of the fourth term.

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6. (a) The first term of an arithmetic series is 10 and the common difference is 3. Calculate

(i) $T_3$, the third term

(ii) $S_3$, the sum of the first 3 terms.

Show that an expression for the general term, $T_n$, is $7 + 3n$ and for the sum to $n$ terms, $S_n$, is $\frac{17n + 3n^2}{2}$.

(b) (i) A sum of IR£P on deposit in a bank earns compound interest at 8% per annum.
Write down an expression, involving P, for the total amount after 2 years.

(ii) A sum of IR£P on deposit in a bank earns compound interest at 8% per annum.
At the end of the first year half the amount is withdrawn.
Calculate the amount remaining, in terms of P, at the end of the second year.
Show that your answer in (ii) = half your answer in (i).

7. A hotel offers two different menus for an evening meal — menu A and menu B.
The hotel can cater for a maximum of 50 people on any evening.
The hotel also requires at least 10 people to order menu A and at least 12 people to order menu B.

Menu A costs IR£9 to prepare and menu B costs IR£6 to prepare.
The total preparation costs must not exceed IR£360.

Graph the set showing the possible number of each type of menu which can be ordered.

The profit on menu A is IR£10 and on menu B is IR£7.
Calculate the maximum profit the hotel can make on an evening's meals.

8. (a) Differentiate from first principles $2 - x^2$ with respect to $x$.

(b) Find the equation of the tangent to the curve

$$y = x^3 - x^2 - 5x - 6$$

at the point $(2, -12)$.

(c) The intensity $I$ of the light from a lamp-post is given by

$$I = \frac{10}{r^2},$$

where $r$ is the distance from the lamp-post.

Find the rate of change of $I$ with respect to $r$.

Calculate this rate of change when $r = 3$.