

LEAVING CERTIFICATE EXAMINATION, 1991

MATHEMATICS - ORDINARY LEVEL - PAPER II (300 marks)

FRIDAY, 7 JUNE - MORNING, 9.30 to 12.00

Attempt **QUESTION 1** (100 marks) and **FOUR** other questions (50 marks each)

Marks may be lost if all your work is not clearly shown
or if you do not indicate where a calculator has been used

1. (i) A salesperson is paid a salary of IR£10 000 per annum plus a commission of 5% on sales. If the value of the sales for a year is IR£200 000, calculate the salesperson's gross pay.

$$100\% = 200,000 \quad 5\% = 200,000 \div 5 + 10,000$$

- (ii) The sum of three consecutive odd natural numbers is 999. Find the largest number.

(iii) Solve $3(x - 2)^2 - 7 = 7(1 - x)$.

- (iv) Show that $x = 4$ is a root of

$$x^3 - 2x^2 - 11x + 12 = 0.$$

- (v) Solve the simultaneous equations

$$2x + 3 = 8 - y$$

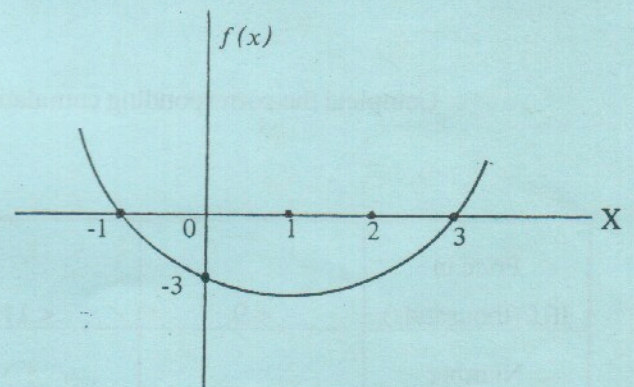
$$\frac{x}{y} = 2$$

- (vi) The general term, T_n , of an arithmetic sequence is $3 - 5n$.

Verify that $T_{n+1} - T_{n-1} > T_3$.

- (vii) The graph of the quadratic function $x \rightarrow f(x)$, $x \in \mathbf{R}$, is as shown.

Express $f(x)$ in the form $x^2 + bx + c$.



- (viii) If $5^{2x+1} = \frac{125}{\sqrt{5}}$, find the value of x .

- (ix) The function f is defined by

$$f: \mathbf{R} \rightarrow \mathbf{R}: x \rightarrow 2x - 3.$$

For what value of t is $f(t) + f(2t) = 0$?

- (x) Differentiate $(x^2 - 5x)^6$ with respect to x .

2. Let $z = 3 - 2i$, where $i = \sqrt{-1}$. Calculate z^2 and $\frac{13}{z}$.

Plot i , z and $\frac{13}{z}$ on an Argand Diagram.

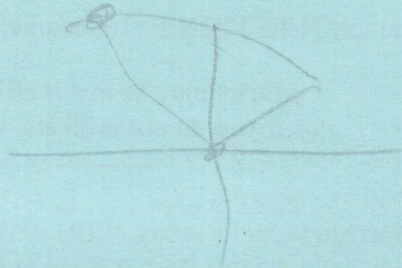
$z = 3 - 2i$ is a solution of the equation

$$z^2 + bz + c = 0, \quad b, c \in \mathbf{R}.$$

Find the value of b and the value of c .

Investigate if $|z| = |iz|$.

Give a geometrical interpretation for your answer.



3. A survey showed the price of new cars bought in a given area over one month as follows:

Price in IR£ (thousands)	6—9	9—11	11—13	13—15	15—20
Number of cars	30	40	50	50	40

(Note 6 - 9 means 6 is included but 9 is not etc.).

Draw a histogram to represent the data.

How many new cars were bought.

Calculate the mean (average) price of a new car, taking the prices at the mid-interval values.

Complete the corresponding cumulative frequency table:

Price in IR£ (thousands)	< 9	< 11	< 13	< 15	< 20
Number of cars	30	70	120	170	210

Draw the cumulative frequency curve.

Use the curve to estimate

(i) the median price per car

(ii) the number of cars costing more than IR£ 14 000.

4. If $f(x) = 2x^3 - 3x^2 - 3x + 2$, complete the following table :

$-2 - 3 + 3 + 2$

x	-1.5	-1	-0.5	0	0.5	1	1.5	2	2.5
$f(x)$	-7	0	2.5	2	0	-8	-2.5		7

Draw the graph of the function

$$f: x \rightarrow 2x^3 - 3x^2 - 3x + 2$$

in the domain $-1.5 \leq x \leq 2.5$.

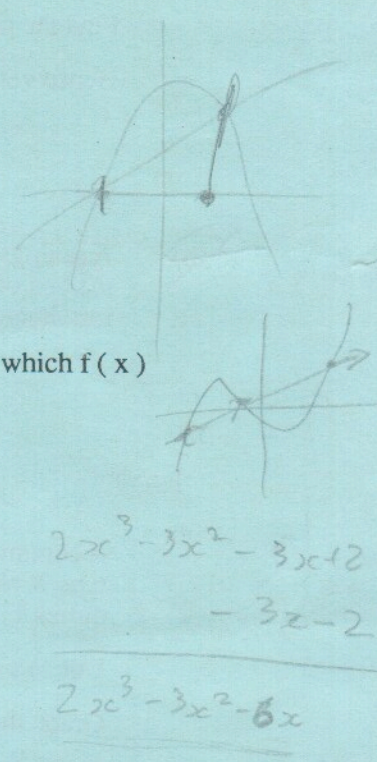
Use your graph to find, as accurately as possible, the range of values of x for which $f(x)$ is an increasing function.

Using the same axes and the same scales draw the graph of the function

$$g: x \rightarrow 3x + 2, \text{ for } x \in \mathbb{R}.$$

Using both graphs, or otherwise, find the range of values of x for which

$$2x^3 - 3x^2 - 6x > 0.$$



5. (a) Solve $\frac{5}{x-1} = \frac{1}{2} - \frac{x+3}{2x}$

(b) Verify that $\begin{pmatrix} 4 \\ 0 \end{pmatrix} + \begin{pmatrix} 4 \\ 2 \end{pmatrix} + \begin{pmatrix} 4 \\ 4 \end{pmatrix} = \begin{pmatrix} 4 \\ 1 \end{pmatrix} + \begin{pmatrix} 4 \\ 3 \end{pmatrix}$

(c) Expand in descending powers of x

$$(3x - 1)^4.$$

Using the expansion find the value of x for which

$$81x^4 + 54x^2 + 1 = 108x^3 + 12x.$$

6. (a) Write out the first three terms of the arithmetic series

$$S_n = n^2 - 3n,$$

where S_n is the sum to n terms. Hence find the value of a and the value of d .

Verify that $S_{n+3} - S_n = 6n$.

Find the general term T_n .

Hence verify that for $n = 1$

$$S_{n+3} - S_n = T_5.$$

(b)

A savings fund pays 10% per annum compound interest. A person pays IR£2000 into the fund. At the end of the first year he pays in another IR£1800. He wishes to withdraw his money in two equal instalments of IR£ W at the end of the second year and the third year. Calculate IR£ W .

9. A community workshop produces two types of articles, X and Y. During production, an article of type X requires 3 minutes on machine A and 4.5 minutes on machine B; an article of type Y requires 1.5 minutes on machine A and 1 minute on machine B.

Due to overheating, each machine cannot operate for more than 90 minutes each day.

Graph the set showing the possible number of each type of article that can be produced each day.

The profit on the sale of an article of type X is IR£ 2.50 and for an article of type Y is IR£1.

How many of each type of article should be produced each day so that the profit is a maximum.

What is the maximum profit?

8. (a) Differentiate from first principles $3x^2 - 5x$ with respect to x .

- (b) Find the coordinates of the local maximum and the local minimum of the curve

$$y = x^3 - 48x + 8.$$

- (c) The volume V of a certain gas is given by $V = \frac{600}{p}$, where p is the pressure.

Find the rate of change of V with respect to p . Calculate this rate of change when $p = 20$.