60462

M.47

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LEAVING CERTIFICATE EXAMINATION, 1991

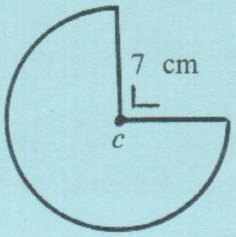
MATHEMATICS – ORDINARY LEVEL – PAPER I (300 marks)

THURSDAY, 6 JUNE – MORNING, 9.30 - 12.00

Attempt QUESTION 1 (100 marks) and FOUR other questions (50 marks each)

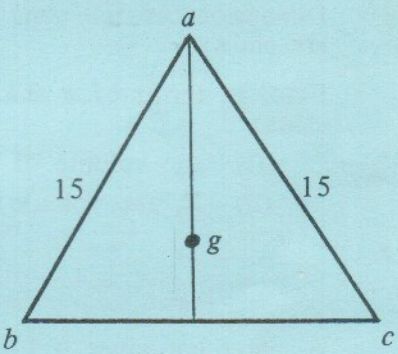
Marks may be lost if all your work is not clearly shown
or if you do not indicate where a calculator has been used.

1. (i) A disc, centre c, radius 7 cm has a quarter sector removed.
Calculate the length of the perimeter taking $\pi = \frac{22}{7}$.

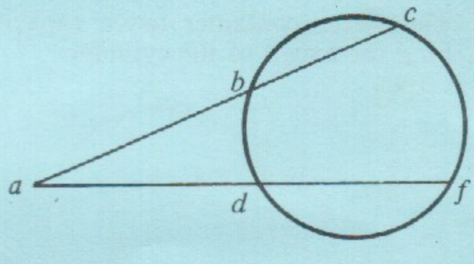


(ii) If $\frac{r}{v} + \frac{r}{k} = p$, express r in terms of p, v and k.

(iii) In the triangle abc, $|ab| = |ac| = 15$ cm.
g is the point of intersection of the medians (centroid). $|ag| = 8$ cm.
Calculate $|bc|$.



(iv) In the diagram $|ab| = 8$ cm, $|bc| = 6$ cm, and $|ad| = 7$ cm.
Calculate $|df|$.



(v) The distance between points $(t, -1)$ and $(2, 2)$ is 5. Find two values for t.

(vi) Find the equation of the line containing $(3, -4)$ and parallel to $2x - 5y = 10$.

(vii) S is the circle $x^2 + y^2 = 16$.
Write the equation of the circle centre $(0, 0)$ which has an area four times that of S.

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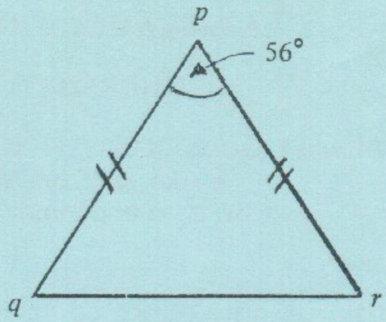
1 (contd.) (viii) For what value of θ, $0° < \theta < 90°$ is

$$2 \sin\theta \cos\theta = \sin\theta \ ?$$

(ix) The area of the isosceles triangle pqr is $6{\cdot}632 \text{ cm}^2$.
Calculate $|pr|$. (Tables P.17).



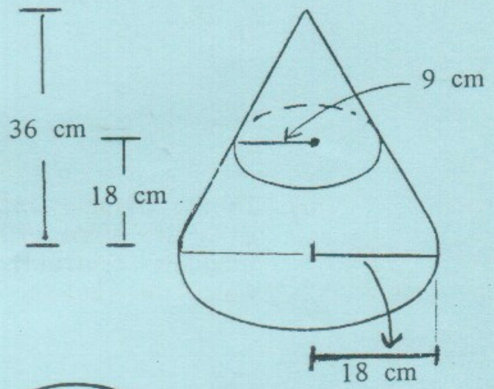
(x) $\vec{v} = 7\vec{i} - 7\vec{j}$, $\vec{uv} = 3\vec{i} + 4\vec{j}$.
Find $\vec{u}$ in terms of $\vec{i}$ and $\vec{j}$.

2. A cone has the upper part removed by a cut which is parallel to the base.
Dimensions of the cone and the remainder are shown.

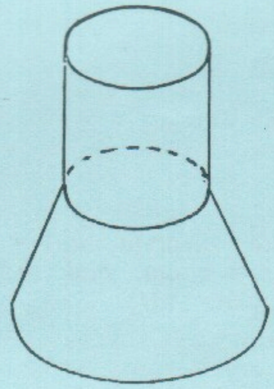
Find, in terms of π (i.e. not giving π a value)

(i) the volume of each cone

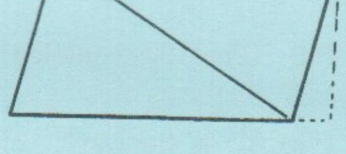
(ii) **R**, the volume of the remainder.



A water container is formed by fixing an open cylinder to the remainder.
The water container has a capacity $\frac{3}{2}$ **R**
Find the height of the cylinder.

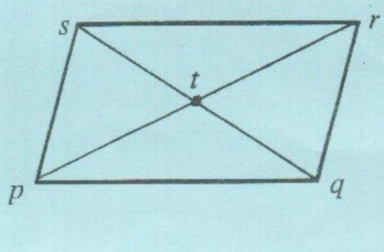


3. (i) Prove that the areas of two triangles of equal height are proportional to the lengths of their bases.

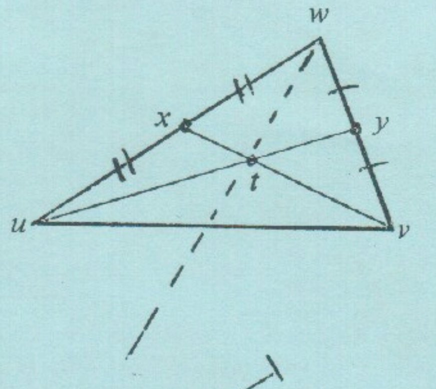


(ii) Hence, or otherwise, prove that a diagonal bisects the area of a parallelogram.

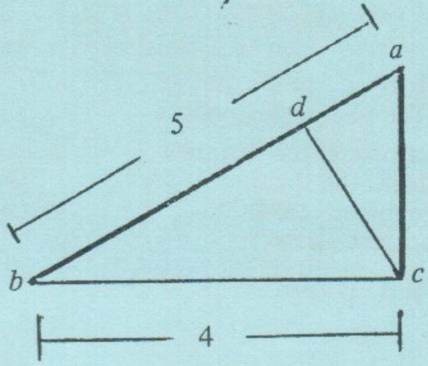
(iii) Assuming the diagonals of a parallelogram bisect one another, prove that the diagonals divide the parallelogram $pqrs$ into four triangles of equal area.



4. (a) Prove that the medians of a triangle are concurrent.



(b) In the triangle shown, $|\angle bca| = |\angle cdb| = 90°$
Prove the triangles, *bcd*, *bca* are equiangular.
Calculate $|cd|$ if $|ab| = 5$ and $|bc| = 4$.



5. H is the line $3x + 2y - 4 = 0$.
Verify that $c(2, -1)$ is in H.

Points $a(-5, 1)$ and $b(1, 9)$ are in L.
Find

(i) the equation of L.

(ii) the coordinates of d, the point of intersection of H and L.

(iii) the coordinates of the fourth point m of the parallelogram *dacm*.

(iv) the area of *dacm*.

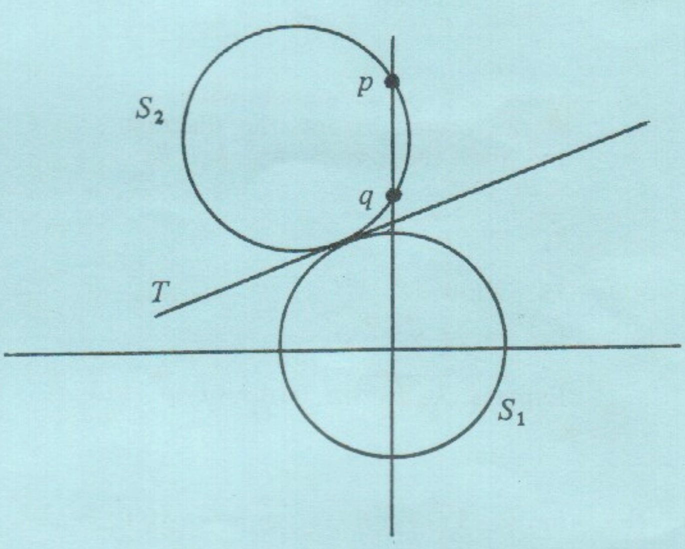
6. Write down the length of the radius of the circle

$$S_1 : x^2 + y^2 = 20.$$

$T : x - 2y + 10 = 0$ is the equation of a tangent to S_1.
Find the coordinates of the point of contact.

S_2 is the image of S_1 under an axial symmetry in T. Write the equation of S_2.
Find the coordinates of p and q, points in which S_2 intersects the Y axis.

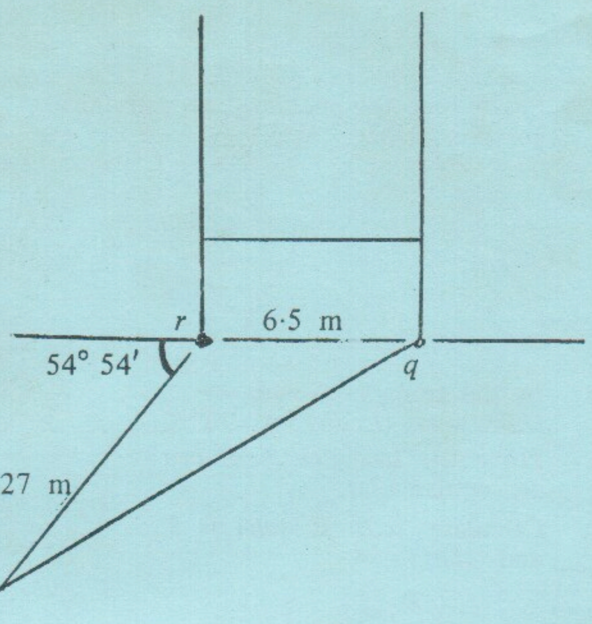
S_3 is a circle through p and q with centre $(6, 8)$
Find the equation of S_3.



7. (a) A ball at p is 27 m from the nearer goalpost.

(i) Calculate its distance from the farther goalpost, to the nearest metre.

(ii) Find $|\angle rpq|$.



(b) Sketch the graph of $\cos 2x$ in the domain $0 \leqslant x \leqslant \pi$.
Use the graph to estimate the values of x for which $\cos 2x = 0{\cdot}7$.

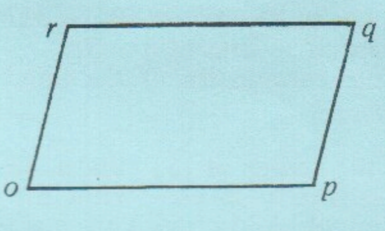
8. (a) $opqr$ is a parallelogram with o the origin. Copy the diagram and plot the points k_1, k_2, k_3 such that

$\vec{k_1} = \vec{p} + \vec{q}$

$\vec{k_2} = \vec{p} + \frac{1}{2}\vec{r}$

$\vec{k_3} = \frac{1}{2}\vec{p} + \vec{r}$.

Express $\overrightarrow{k_2 k_3}$ in terms of $\vec{p}$ and $\vec{r}$.



(b) $\vec{m} = 5\vec{i} - 6\vec{j}$, $\vec{n} = -10\vec{i} + 2\vec{j}$.
Find $\overrightarrow{mn}$ in terms of $\vec{i}$ and $\vec{j}$.

If $\vec{m} + \frac{1}{2}\overrightarrow{mt} = \vec{n}$, find $\vec{t}$ in terms of $\vec{i}$ and $\vec{j}$.